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## ABSTRACT

### Estimating Regional Fluxes of Evapotranspiration and Sensible Heat From Measurements of the Planetary Boundary Layer

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A study was conducted to estimate regional fluxes of evapotranspiration and sensible heat using Planetary Boundary Layer Similarity Theory and a Conservation Equation Approach. Measurements were made of dry-bulb temperature, humidity, and pressure throughout the planetary boundary layer (PBL) using rawinsondes. The measurements were taken over the Konza Prarie Natural Research Area, near Manhattan, Kansas from 21-24 July, 1986.

Soundings of virtual potential temperature were plotted to determine the height of the PBL. The estimates of heat flux were compared to a set of nine surface measurements.

Estimates using the Conservation Equation Approach fell within one standard deviation of the mean measured values in four of five cases for both evapotranspiration and sensible heat flux on 22 July. Soundings for 23 July were not of good quality due to many missing points and yielded estimates for both evapotranspiration and sensible heat flux which were not very reasonable.

Estimates of sensible heat flux calculated using the PBL Similarity Theory were not very reasonable. Estimates were generally below the mean measured values.

Evapotranspiration flux estimates agreed better with the mean surface measurements for the PBL Similarity Theory using values at the top of the PBL when the quality of the soundings was good. Estimates were

within one standard deviation for six of the fourteen soundings. Vertically averaged values performed better when the soundings were poor.

The Conservation Equation Approach used yielded better agreement with the ground truth measurements for both evapotranspiration and sensible heat flux.

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ESTIMATING REGIONAL FLUXES OF EVAPOTRANSPIRATION AND SENSIBLE HEAT  
FROM MEASUREMENTS OF THE PLANETARY BOUNDARY LAYER

by

William G. Munley Jr., Captain, USAF

A thesis submitted in partial fulfillment  
of the requirements for the degree

of

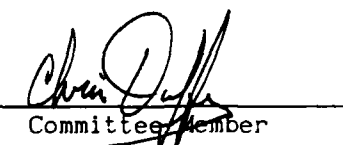
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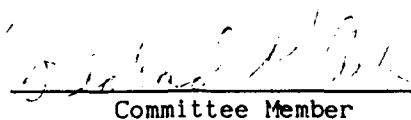
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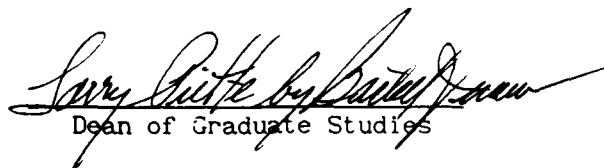
Soil Science and Biometeorology

Approved:

  
Major Professor

  
Committee Member

  
Committee Member

  
Dean of Graduate Studies

UTAH STATE UNIVERSITY  
Logan, Utah

1988

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THANKS ALSO TO THE BOYS!

William Gregory Munley Jr.

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## ABSTRACT

Estimating Regional Fluxes of Evapotranspiration and Sensible Heat  
From Measurements of the Planetary Boundary Layer

by

William G. Munley Jr., Master of Science

Utah State University, 1988

Major Professor: Dr. Lawrence E. Hipps  
Department: Soil Science and Biometeorology

A study was conducted to estimate regional fluxes of evapotranspiration and sensible heat using Planetary Boundary Layer Similarity Theory and a Conservation Equation Approach. Measurements were made of dry-bulb temperature, humidity, and pressure throughout the planetary boundary layer (PBL) using rawinsondes. The measurements were taken over the Konza Prairie Natural Research Area, near Manhattan, Kansas from 21-24 July, 1986.

Both theoretical approaches require the height of the planetary boundary layer as an input to the equations. Soundings of virtual potential temperature were plotted to determine the height of the inversion. The height of the PBL ranged from 275 meters in the morning to 1700 meters when fully developed in the afternoon over the four-day period of this study. The estimates of evapotranspiration and sensible heat flux were compared to a set of nine surface measurements.

Estimates for July 22 were based on good quality soundings. On that day estimates using the Conservation Equation Approach fell within

one standard deviation of the mean measured values in four of five cases for both evapotranspiration and sensible heat flux. Soundings for July 23 were not of good quality due to many missing points and yielded estimates for both evapotranspiration and sensible heat flux which were not very reasonable.

Estimates of sensible heat flux calculated using the PBL Similarity Theory were not very reasonable. Using both values at the top of the PBL and vertically averaged values yield results within one standard deviation of the mean measured values for only three of the fourteen soundings. Estimates were generally below the mean measured values.

Evapotranspiration flux estimates agreed better with the mean surface measurements for the PBL Similarity Theory using values at the top of the PBL when the quality of the soundings was good. Estimates were within one standard deviation for six of the fourteen soundings. Vertically averaged values performed better when the soundings were poor. Overall using temperature and humidity values at the top of the PBL gave more acceptable estimates of evapotranspiration flux.

The Conservation Equation Approach used yielded better agreement with the ground truth measurements for both evapotranspiration and sensible heat flux.

## INTRODUCTION

To understand properly the large-scale interactions between the earth's surface and the atmosphere requires addressing the regional scale exchanges of mass and energy across this interface. Latent heat flux or evapotranspiration (ET), the sum of evaporation from soil or water and transpiration from vegetation, is often the largest energy exchange in the surface-atmosphere interface. Estimates of ET are very important in hydrology for use in stream flow forecasting, ground water recharge estimates, and general water management. Agriculture uses ET estimates for forecasting plant water supplies, irrigation system design, crop pattern planning, and soil moisture deficit mapping for crop yield prediction models. In meteorology, regional ET estimates are required for calculations in large-scale atmospheric circulation models. Any rational examination of land surface climatology clearly must consider large-scale latent heat flux.

The literature contains a wide variety of techniques for estimating ET, both soil-related and meteorological, but most of them are of minimal value for estimating ET on a regional scale. Most of the existing techniques are designed for use on individual field scales or point estimates and yield only local estimates of ET. In order to address the climatology of a larger area of land surface, such as an ecosystem, one must deal with ET estimates on a regional scale on the order of 100-10,000 km<sup>2</sup>.

A study of the ET process requires an in-depth look at the layer of the atmosphere which is directly influenced by the dynamic effects of the surface. This layer is called the planetary boundary layer (PBL).

Although the thickness of the PBL is ever-changing, it has a typical depth of 1 km when fully developed for a neutral atmosphere. Since the structure of the PBL is strongly determined by the large-scale surface properties, it is anticipated that the regional surface energy and mass fluxes can be determined from knowledge of the properties and depth of the PBL.

Two theoretical approaches have recently been developed which use information from the PBL to estimate regional evapotranspiration and sensible heat flux. The first approach can be termed PBL Similarity Theory. This theory evolved over many years and has its roots in the Navier-Stokes and continuity equations. The theory defines the relationship between properly scaled external parameters (at the top of the PBL) and surface fluxes of momentum, sensible heat, and latent heat. It is assumed that the PBL is composed of two layers: a surface layer where mean winds, temperature, and humidities are functions of the surface fluxes and surface properties; and an outer layer, or Ekman Layer, where the mean values are functions of surface fluxes but exhibit no dependence on the surface properties (ie. roughness).

The second approach is referred to as the Conservation Equation Approach. McNaughton and Spriggs (1986) proposed the Mixed Layer PBL Model. In this model the growth of the PBL is simulated and the PBL itself is broken up into three layers. The first two are the same as the layers used in the PBL similarity theory while the third is a thin entrainment layer above the PBL. After the growth of the PBL is simulated the conservation equations for temperature and humidity are solved. The Conservation Equation Approach simply solves these

equations without the need for simulating the PBL.

These two approaches seem to be the most scientifically sound of the regional flux approaches proposed, but a thorough verification experiment has yet to be run on them. This study will examine the feasibility of estimating regional evapotranspiration and sensible heat flux using these two theoretical approaches. Data were collected from balloon soundings of temperature, humidity and pressure made over the Konza prairie in Kansas during a four-day period in July 1986.

More specifically, the objectives of this study are as follow:

- I. Evaluate the structure and depth of the PBL for each sounding.
- II. Using the data from objective I, calculate regional ET and sensible heat estimates for each sounding using the two theoretical approaches previously introduced.
- III. Compare the estimated ET and sensible heat values against ground-based measurements of ET and sensible heat measured by nine Bowen Ratio systems at numerous sites on the Konza prairie.
- IV. Determine which of the two theoretical methods yields the most accurate estimates of ET and sensible heat.

## THEORETICAL CONSIDERATIONS

To simply state the proposed approaches for estimating evapotranspiration and sensible heat and give a brief description of them does not do justice to the theory behind them. In order to fully understand the approaches evaluated in this paper, a description of the development of the pertinent equations is necessary.

### PLANETARY BOUNDARY LAYER SIMILARITY THEORY

The first approach considered is often termed PBL Similarity Theory. This approach relates properly scaled external parameters (at the top of the PBL) with surface fluxes of momentum, sensible heat, and latent heat.

The equations at the root of this theoretical approach are the equation of continuity (conservation of mass) and the nonlinear Navier-Stokes equations for momentum.

Derivation of the conservation of mass equation yields (Landau and Lifschitz, 1959):

$$\partial \rho / \partial t + \partial (\rho u_i) / \partial x_i = 0 \quad (1)$$

where  $\rho$  is the density of air,  $t$  the time, and  $u_i$  represents the components of the wind velocity. This equation can also be written as (Businger, 1982):

$$\partial u_i / \partial x_i = -1/\rho \partial \rho / \partial t \quad (2)$$



If we assume that the fluid is incompressible then (2) becomes:

$$\partial u_i / \partial x_i = 0 \quad (3)$$

This is the form of the continuity equation that is commonly used in atmospheric science.

The second and more complex set of equations are the Navier-Stokes equations for momentum. This set of equations was completed by Stokes in 1845 and remains the standard for the study of momentum transfer. The three equations which describe the momentum transfer in three dimensions are commonly written in tensor notation as a single equation (Batchelor, 1967):

$$\begin{aligned} \partial(\rho u_i) / \partial t + \partial(\rho u_j u_i) / \partial x_j = & -\partial P / \partial x_i + \partial / \partial x_j \\ & [\mu (\partial u_i / \partial x_j + \partial u_j / \partial x_i) + 2/3 \mu \partial u_k / \partial x_k \sigma_{ij}] \\ & + \rho g_i - 2 \varepsilon_{ijk} \Omega_j u_k \end{aligned} \quad (4)$$

where  $P$  is the pressure,  $\mu$  is the dynamic viscosity,  $\Omega$  is the angular velocity of the earth's rotation,  $g_i = (0, 0, -g)$  or the gravitational acceleration in the  $z$  or  $k$  component of the equation, and  $i, j, k$  are an alternating tensor to allow for the coriolis term to be present in each equation. The first term on the left side of the equation represents the local rate of change of momentum while the second term is the advection of momentum. On the right side the first term is the pressure gradient, next is the viscous force, third is the gravitational component, and finally there is a term for reference frame adjustment usually called the Coriolis effect.

In order to get this equation into a form that is usable in the normal turbulent atmosphere, certain assumptions are generally made. First, it is assumed that the dynamic viscosity ( $\mu$ ) and the molecular heat conductivity are constant throughout the fluid. Next, a reference atmosphere is defined which exists in adiabatic and hydrostatic equilibrium, having temperature, pressure, and density  $T_0$ ,  $p_0$ , and  $\rho_0$ , respectively. If  $\rho' = \rho - \rho_0$ , where  $\rho$  is the actual density, then it is assumed that  $\rho'/\rho_0 \ll 1$ . A similar assumption for temperature and pressure are made so that  $T'/T_0 \ll 1$  and  $p/p_0 \ll 1$  where  $T' = T - T_0$  and  $p = p - p_0$ . By definition  $\partial T_0 / \partial x_3 = -g/c_p$ , where  $c_p$  is the specific heat at constant pressure. Finally it is assumed that the vertical scales of motion are small relative to the scale height (Busch, 1973).

Using these assumptions Dutton and Fichtl (1969) derived a set of equations for use in both shallow and deep convection atmospheres. Busch (1973, p. 3) stated:

The Boussinesq equations appear to be a natural step in the progression of theory from inviscid incompressible flows, through viscous incompressible flows to viscous compressible flows. In the Boussinesq approximations all flows are treated as incompressible but with a temperature dependent density, the variation of which is significant only when multiplied by the acceleration due to gravity.

This being the case, (3) is valid and the momentum equations become:

$$\begin{aligned} \partial u_i / \partial t + u_j \partial u_i / \partial x_j = & -1/\rho_0 \partial P / \partial x_i + \nu \partial^2 u_i / \partial x_j^2 \\ & + g T' / T_0 \sigma_{1,3} - 2\epsilon_{ijk} \Omega_j u_k \end{aligned} \quad (5)$$

where the kinematic viscosity  $\nu = \mu/\rho$ .

Now that the basic equations have been established they must be adapted so that they can be applied to mean flow calculations.

Dealing with mean atmospheric flow is not as simple as assuming a laminar flow field. The real atmosphere always exhibits fluctuations in the velocity, temperature, and pressure fields (Fig 1).

To handle these fluctuations the flow is divided into a temporal mean and a deviation. Reynolds averaging is then employed to yield the Reynolds Stress equations. The Reynolds convention is equivalent to splitting the flow field into a mean flow and a fluctuating part, where the average of the fluctuations is zero (Busch, 1973):

$$\begin{aligned} u_i &= \bar{u}_i + u_i' , & \bar{u}_i' &= 0 \\ P &= \bar{P} + p' , & \bar{p}' &= 0 \end{aligned} \quad (6)$$

This convention uses the overbar to represent a time average and a prime to show the fluctuation value. Applying this averaging process to (3) and (5) yields the Reynolds stress equations:

Continuity Equation:

$$\partial \bar{u}_i / \partial x_i = 0 \quad (7)$$

Momentum Equations:

$$\begin{aligned} \partial u_i / \partial t + u_j \partial \bar{u}_i / \partial x_j &= -1/\rho_o \partial P / \partial x_j + \nu \partial^2 \bar{u}_i / \partial x_j^2 \\ + g T' / T_o \sigma_{i,3} - 2\Omega \epsilon_{ijk} \bar{u}_k - \partial / \partial x_i (\overline{u_i' u_j'}) \end{aligned} \quad (8)$$

where the final term on the right side is the shear divergence.

If the assumption of horizontal homogeneity and steady state are made,  $\partial / \partial t = \partial / \partial x = \partial / \partial y = 0$  and the continuity equation becomes:

$$\partial \bar{w} / \partial z = 0 \quad \text{or} \quad \bar{w} = 0 \quad (9)$$

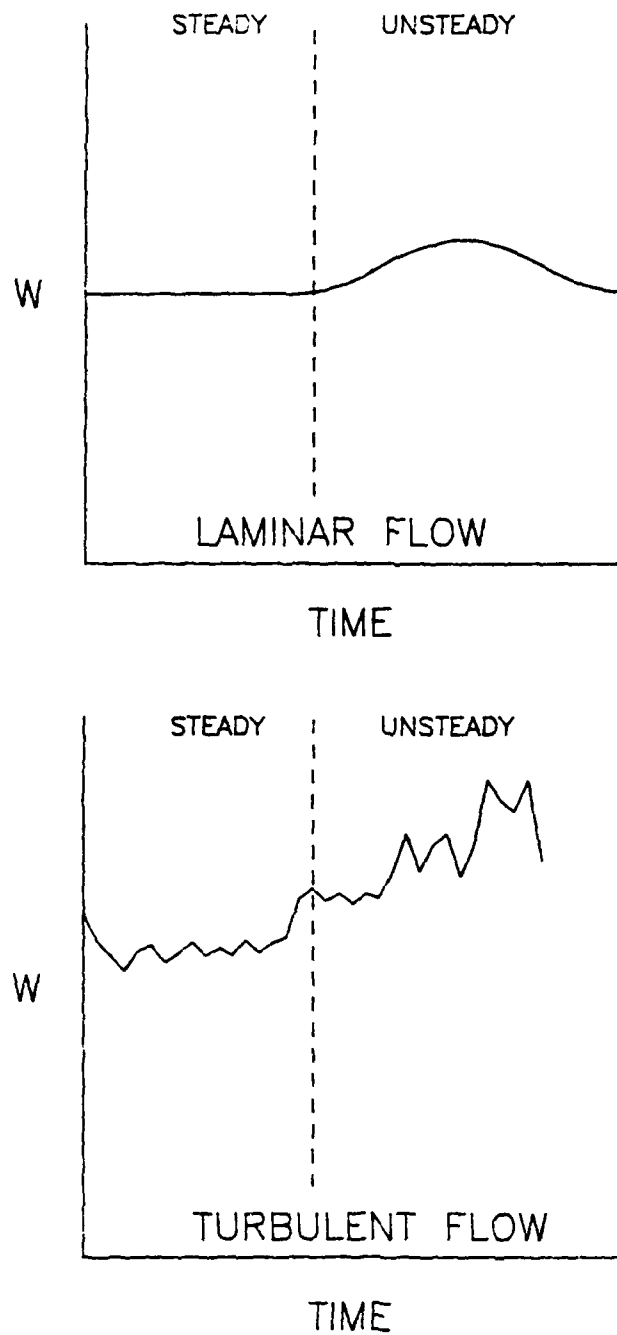


Fig. 1. Comparison of laminar and turbulent flow showing the fluctuations of vertical windspeed ( $w$ ) with respect to time in the steady and unsteady cases.

The Reynolds equations for the averaged quantities are reduced to (Businger, 1982):

$$\partial(\overline{u'w'})/\partial z = f(v-v_g) + \nu \partial^2 \overline{u}/\partial z^2 \quad (10)$$

$$\partial(\overline{v'w'})/\partial z = -f(u-u_g) + \nu \partial^2 \overline{v}/\partial z^2 \quad (11)$$

where  $f$  is the Coriolis parameter  $f = 2\Omega \sin\phi$ , and  $u_g$  and  $v_g$  are the horizontal components of the geostrophic wind, defined as the horizontal wind velocity where the Coriolis acceleration ( $f$ ) exactly balances the horizontal pressure force.  $u$  and  $v$  are the horizontal components of velocity,  $\overline{u'w'}$  and  $\overline{v'w'}$  are the vertical fluxes of horizontal momentum, and  $z$  is the height. The last term on the right sides of the equations represents the contribution of viscous forces.

Now that the governing equations are in a form that can be readily used, it is necessary to find the appropriate length and velocity scales for the PBL.

The areas of interest are the properties of the PBL at large values of  $z/z_0$ , where  $z$  is the height and  $z_0$  is the surface roughness. If  $z_0$  is used to scale the layer then large-scale features will be lost. On the other hand, if  $h$  is used as the scaling factor then the small-scale features will be lost. To properly scale the PBL, the layer is often broken into two separate layers. These layers are the thin surface layer and the outer layer, or the Ekman layer (Fig. 2) (Blackadar and Tennekes, 1968).

This scaling process isn't as easy as it may seem. First, the wind in the surface layer ( $U$ ) is assumed to be on the same order of magnitude as the geostrophic wind ( $G$ ). Thus the ratio of  $U/G$  is about

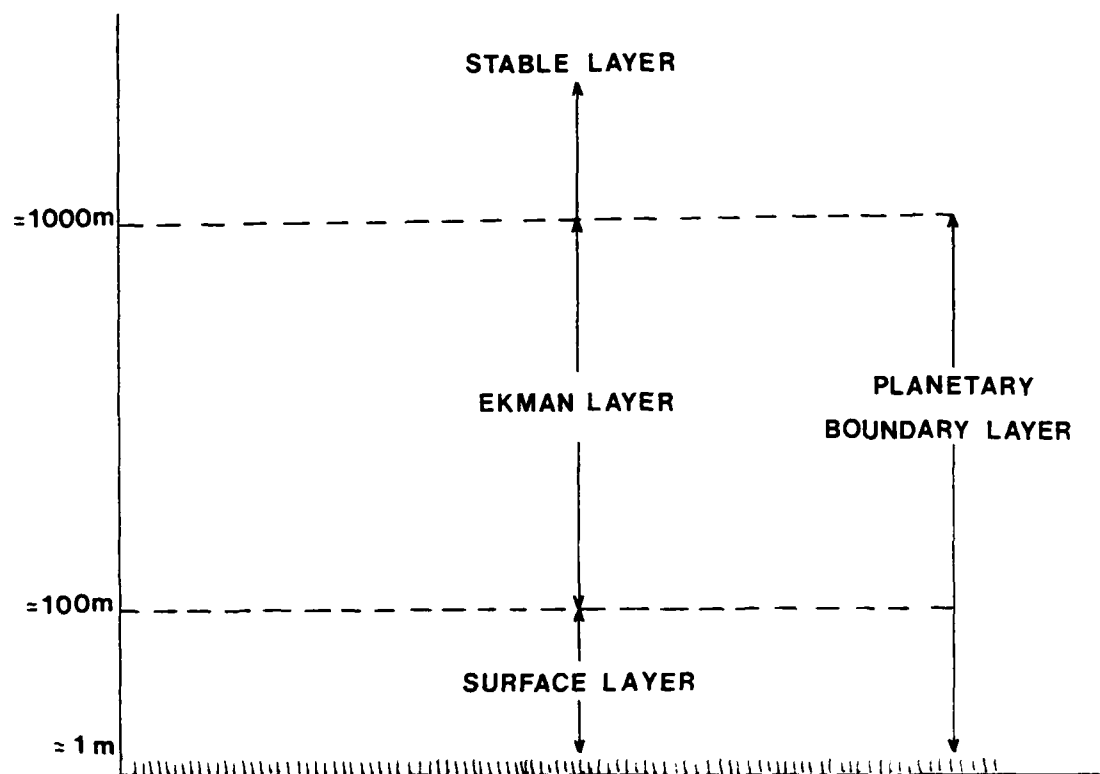


Fig. 2. Typical convective boundary layer showing distinct layers with order of magnitude height values.

one, and  $G$  seems like the correct scaling velocity for the wind near the surface. Before that can be assumed reasonable we must consider the fact that the wind near the surface is a function of height. This brings up another question. What height scale should be used?

There are two heights which are possible to scale the wind. First, the surface roughness ( $z_0$ ), which is small, and  $G/f$ , which is large.  $G/f$  is related to the synoptic pressure gradient and the presence of ageostrophic velocity components in the PBL. Ageostrophic velocity components are defined as the vector difference between the real wind and the geostrophic wind. Combining the two heights gives us the surface Rossby number,  $G/(fz_0)$ , which is a central nondimensional parameter in PBL flow (Tennekes, 1982).

Noting that the surface stress ( $\tau_0$ ) is a function of the Rossby number we can state that the characteristic velocity scale of turbulence based on the surface stress is the friction velocity ( $u_*$ ). The surface stress is then defined by:

$$\tau_0 = \rho u_*^2 \quad (12)$$

If we say that  $\tau_0$  is a function of the Rossby number then  $u_*$  must also be a function of  $G/(fz_0)$ . The relationship is given as:

$$u_*/G = F_g (G/fz_0) \quad (13)$$

The question then arises, Which of the velocity scales,  $u_*$  or  $G$ , is appropriate for use in the PBL? Tennekes (1982, p. 39) stated,

The friction velocity is not an independent, external parameter as far as the flow in the planetary boundary layer is concerned. However, it does account for the effects of the large-scale pressure field and the surface roughness. Also, since the surface stress equals the turbulent momentum flux in the air just above the surface,  $u^*$  is in some sense representative of the turbulent wind fluctuations in the lower layers. Finally, since the surface wind is observed to be aligned with the surface stress the use of  $u^*$  as a velocity scale does not involve any problems that arise from the difference in direction between  $\tau_0$  and  $G$ .

Using  $u_*$  as the velocity scale and  $z_0$  as the height scale we can write the wind profile in a neutral surface layer as:

$$U/u_* = f_x(z/z_0) \quad (14)$$

where  $U$  is the average wind in the surface layer and  $f_x$  represents a function in the  $x$  direction.

If the wind profiles are plotted in this manner then the wind will show explicit dependence on the surface Rossby number. In this situation  $u_*$  is a dependent internal parameter of the PBL and (14) is based on internal parameterization. The problem is not solved until  $F_g$  in (13) and  $f_x$  in (14) are found.

Equations (10) and (11) represent the equations of motion for steady, horizontally homogeneous turbulent flow in a neutrally stratified atmosphere. Since the scale of the PBL is large the viscous terms in (10) and (11) are neglected. By aligning the  $x$ -axis in the same direction as the shearing stress ( $\tau_0$ ), when  $z = z_0$  we get (Tennekes, 1982):

$$\tau_x = -\overline{u'w'} = \tau_0 = u_*^2 \quad (15)$$

$$\tau_y = -\overline{v'w'} = 0 \quad (16)$$



Using these boundary conditions the Reynolds stress components in (10) and (11) are nondimensionalized using the surface stress. By doing this the normalized stresses,  $\overline{u'w'}/u_*^2$  and  $\overline{v'w'}/u_*^2$ , remain finite regardless of how large the Rossby number becomes.

Next, a normalizing velocity for the velocity difference  $(u - u_g)$  and  $(v - v_g)$  must be found. Tennekes and Lumley (1972) found through laboratory studies that  $u_*$  is the proper velocity scale for the ageostrophic wind components. Using this reasoning equations (10) and (11) are transformed into the nondimensional set:

$$(v - v_g)/u_* = [d(\overline{u'w'})/u_*^2]/[d(zf)/u_*] \quad (17)$$

$$(u - u_g)/u_* = [d(\overline{v'w'})/u_*^2]/[d(zf)/u_*] \quad (18)$$

where  $v_g$  and  $u_g$  are components of the geostrophic wind. If the surface boundary conditions are ignored equations (17) and (18) yield the velocity defect laws for the Ekman layer (Blackadar and Tennekes, 1968):

$$(u - u_g)/u_* = F_x(zf/u_*) \quad (19)$$

$$(v - v_g)/u_* = F_y(zf/u_*) \quad (20)$$

where  $F_x$  and  $F_y$  are the universal velocity defect functions.

In the surface layer the roughness height ( $z_0$ ) is the appropriate length scale and  $u_*$  is the velocity scale. Using these scales, (10) and (11) become (Tennekes, 1982):

$$-(fz_0/u_*) [(v - v_g)/u_*] = -[d(\overline{u'w'})/u_*^2]/(dz/z_0) \quad (21)$$

$$(fz_0/u_*) [(u - u_g)/u_*] = - [d(\overline{v'w'})/u_*^2]/(dz/z_0) \quad (22)$$

The left side of these equations is very small and as  $G/(fz_0) \rightarrow \infty$  they become 0. Considering the boundary conditions (15) and (16) to be valid, the shear stress is about constant for all values of  $z/z_0$  that are not large (Tennekes, 1982):

$$-\overline{u'w'} = u_*^2 \quad (23)$$

$$-\overline{v'w'} = 0 \quad (24)$$

Restricting the area of interest to the surface layer the similarity law for the surface layer is:

$$U/u_* - f_x (z/z_0) \quad (25)$$

Thus, the scaled velocity defects in the Ekman layer are universal functions of the nondimensional height  $(zf/u_*)$ , while the surface layer velocities are functions of  $(z/z_0)$ . This results in an expression for the Ekman layer which cannot be used close to the surface and a surface expression which is not valid for values of  $z/z_0$  approach the Ekman layer. Assuming that equations (19) and (20) are valid in the Ekman layer and equation (25) applies to the surface layer, there is a small layer in between where both laws are valid.

To solve the problem of missing boundary conditions between the layers, a process known in singular-perturbation theory as asymptotic matching has been used (Van Dyke, 1964). This technique requires that the Ekman layer similarity laws are identical with the surface layer law. In other words, matching is a dual process by which the flow at

the top of the surface layer provides the missing boundary conditions at the bottom of the Ekman layer. Likewise, the Ekman layer provides upper boundary conditions for the surface (Blackadaar and Tennekes, 1968).

Using the U component of the wind, the matching process described in Blackadaar and Tennekes (1968) and Tennekes (1982) yields:

$$z/u_* \partial U / \partial z = 1/k \quad (26)$$

when  $z/z_0 \rightarrow \infty$  and  $zf/u_* \rightarrow 0$ . Here, k is the Von Karman "constant", taken in this paper to be 0.41. After integrating (26) we find that there are two alternative forms for this equation (Tennekes, 1982):

$$U/u_* = 1/k \ln(z/z_0) \quad (z/z_0 \gg 1) \quad (27)$$

$$(u-u_g)/u_* = 1/k [\ln(zf/u_*) + A] \quad (zf/u_* \ll 1) \quad (28)$$

where A is a similarity parameter which is a function of the height of the PBL and  $u_*/f$ . The region where these equations are valid is termed the 'inertial sublayer' (Blackadaar and Tennekes, 1968). Taking the difference between equations (27) and (28) gives us :

$$U_g/u_* = 1/k (\ln(u_*/fz_0) - A) \quad (29)$$

A corresponding equation can also be found for the V component in the vicinity of the surface:

$$V_g/u_* = - B/k \quad (30)$$

where B is also a similarity parameter. Defining the amplitude of the geostrophic wind  $G = \sqrt{(U_g^2 + V_g^2)}$  and applying it to equations (29) and

(30), we obtain an implicit relation for the geostrophic drag coefficient  $u_* / G$  as a function of the surface Rossby number  $G / f z_0$  (Tennekes, 1982):

$$G / u_* = 1/k [\{\ln(h/z_0) - A\}^2 + B^2]^{1/2} \quad (31)$$

Up to this point, only a neutrally stratified atmosphere has been considered. Since the atmosphere is rarely neutral, due to buoyancy effects, Rossby number similarity does not provide a very good reference for practical application. To account for the diversion from neutral conditions we apply Monin-Obukhov similarity to the equations.

Monin-Obukhov similarity states that when the atmosphere diverts from neutral conditions the buoyancy effects are a function of the nondimensional height  $z/L$ , where  $L$  is equal to the Monin-Obukhov length given as  $L = -\rho u_*^3 c_p T / g H_0 k$  (Obukhov, 1971).  $L$  is positive during stable conditions and negative for unstable conditions,  $T$  is the temperature at the surface, and  $H_0$  is the surface sensible heat flux.

Deardorff (1970, p. 1209) published results of a numerical study of the PBL that questioned the validity of using  $u_* / f$  as the appropriate height scale for the PBL. He concluded that:

The first result obtained is that even for a very slight degree of thermal instability, namely  $-z_1/L = 4.5$ , the height to which the convection and turbulence extends will not be limited to the height of a neutral PBL, but will be limited only by the average height  $z_1$  of the lowest inversion base." .... "It is therefore concluded that the height  $z_1$  even when time dependent is the most important length scale under nearly all unstable conditions likely to be encountered, with  $u_* / f$  being irrelevant. The relevant indicator of the degree of thermal instability is therefore  $-z_1/L$ , not  $-u_* / (fL)$ .

The subscript 1 in this statement stands for the inversion height.

This argument was further advanced by Deardorff (1972) and

Zilitinkevich and Deardorff (1974).

After the buoyancy and height-scale problems are corrected the problem of baroclinicity must be faced. Arya and Wyngaard (1975) proposed that the baroclinic effects of the atmosphere on similarity parameters can be minimized by using vertically averaged winds and temperatures in the equations. This procedure led to less scatter in the similarity parameters.

Yamada (1976) fit the similarity parameters to the large Wangara data set to come up with numerical values of the functions. By using the vertically averaged winds and temperatures proposed by Arya and Wyngaard (1975) he was able to minimize scatter in the fitted functional forms. He reported that under stable conditions the gradient of the function decreased as the stability increased. After several trials using averaged data he found that the following expressions for the similarity parameters A, B, and C realistically predicted the drag coefficient, the heat transfer coefficient, and the surface wind angle:

$$\begin{aligned}
 A &= \begin{aligned} &1.855 - 0.380 \, h/L && \text{for } 0 \leq h/L \leq 35 \\ &-2.94 \, (h/L - 19.94)^{1/2} && \text{for } 35 < h/L \end{aligned} \\
 B &= \begin{aligned} &3.020 + 0.300 \, h/L && \text{for } 0 \leq h/L \leq 35 \\ &2.85 \, (h/L - 12.47)^{1/2} && \text{for } 35 < h/L \end{aligned} \\
 C &= \begin{aligned} &3.665 - 0.829 \, h/L && \text{for } 0 \leq h/L \leq 35 \\ &-4.32 \, (h/L - 11.21)^{1/2} && \text{for } 35 < h/L \end{aligned}
 \end{aligned}$$

where  $h$  is the height at the top of the PBL. He also reported that, during unstable conditions, as the instability increased the functions approach a constant value. The functions for unstable conditions are:

$$A = 10.0 - 8.145 (1.0 - 0.008376 h/L)^{-1/3}$$

$$B = 3.020 (1.0 - 3.290 h/L)^{-1/3}$$

$$C = 12.0 - 8.335 (1.0 - 0.03106 h/L)^{-1/3}$$

Values for the functions during the neutral stability condition ( $h/L = 0$ ) are  $A = 1.855$ ,  $B = 3.020$ , and  $C = 3.665$ . It must be noted that the final fitted functions  $A, B, C = F(h/L)$  still exhibit a lot of scatter.

A fourth stability function is required to handle the water vapor in the atmosphere. But since the Wangara data set was collected over a dry area where evapotranspiration was assumed to be zero, no information on the D function for water vapor was available.

Brutsaert and Chan (1978) attempted to determine the form of the D function. They used a data set collected over the ocean in the mid-1970s which included radiosonde data and estimates of surface fluxes made using bulk transfer coefficients found by Kondo (1975). They found that D was less than C during the unstable conditions of the study and reported a value of  $D = 0.65C$  for these conditions. This conclusion was consistent with the proposals of Brutsaert and Mawdsley (1976) and Mawdsley and Brutsaert (1977) that the D function was smaller than C.

With these approaches the resulting expressions relating surface fluxes and properties of the PBL are:

$$u_* = G k [(\ln(h_1/z_o) - A)^2 + B^2]^{1/2} \quad (32)$$

$$H_o = -\rho c_p u_* k [\ln(h_1/z_o) - C]^{-1} (\theta_{h_1} - \theta_o) \quad (33)$$

$$LE_o = -\rho u_* k [\ln(h_1/z_o) - D]^{-1} (q_{h_1} - q_o) \quad (34)$$

where A, B, C, and D are similarity parameters which are functions of the non dimensional height ( $h_1/L$ ),  $\theta$  is the potential temperature,  $q$  the specific humidity, and  $LE_o$  is the surface latent heat flux.

To solve equations (32-34) requires determination of the stability ( $h/L$ ) without knowing the surface fluxes,  $u_*$ ,  $H$ , and  $E$ . Solving for  $h/L$  can be accomplished by using an iterative technique and the bulk Richardson number defined by Malgarejo and Deardorff (1974):

$$Ri_b = \frac{gh_1 (\theta_{v_h} - \theta_{v_o})}{T_v G^2} = \frac{1/H_\alpha (h_1/L) [\ln(h_1/z_o - C(h_1/L))]}{\{[\ln(h_1/z_o - A(h_1/L))]^2 + B^2(h_1/L)\}} \quad (35)$$

where  $Ri_b$  is the bulk Richardson number,  $g$  is the gravitational acceleration,  $T_v$  is mean PBL virtual temperature,  $\theta_v$  is the virtual potential temperature with the subscripts h and o referring to the top of the PBL and the surface, respectively,  $G$  is the geostrophic wind, and  $H_\alpha$  is the inverse of the Prandtl or Schmidt number assumed to be 0.74. Once the bulk Richardson number is found, equations (32-34) can be solved to yield estimates for regional scale values of surface fluxes.

#### CONSERVATION EQUATION APPROACH

Another approach for estimating regional evapotranspiration was proposed by McNaughton and Spriggs (1986). This approach uses the equations for the development of a convective planetary boundary layer. The atmosphere in this approach is segregated into three layers instead

of two as in the PBL similarity theory approach. The first two layers, as defined for PBL Similarity Theory, are the surface layer and fully mixed (Ekman) layer. The third layer, an entrainment layer, is considered to be a transition layer at the top of the PBL encompassing portions of the PBL and the stable layer above the PBL (Fig 3). Movement of the air in the fully mixed layer is due to the rise of buoyant plumes produced at the surface. This results in  $\theta$ -profile that is independent of height (Carson, 1973).

The Conservation Equation Approach follows equations for the temperature and humidity budgets presented by Tennekes (1973) and Carson (1973):

$$\rho c_p h \partial \theta_m / \partial t = H + \rho c_p (\theta_s - \theta_m) \partial h / \partial t \quad (36)$$

$$\rho h \partial q_m / \partial t = E + \rho (q_s - q_m) \partial h / \partial t \quad (37)$$

where  $\rho$  is the density of the air,  $c_p$  is the specific heat at constant pressure,  $h$  is the height of the PBL,  $\theta$  the potential temperature,  $t$  is the time,  $q$  is specific humidity,  $H$  is the sensible heat flux, and  $E$  is the latent heat flux. The subscripts  $m$  and  $s$  refer to the mixed layer and the top of the PBL, respectively.

These equations, like those of the PBL similarity theory, have their roots in the Navier Stokes and continuity equations. Concentrating on the heat balance equation, Carson (1973) showed that the work done by Ogura and Phillips (1962), Calder (1968), and Dutton and Fichtl (1969) on the Boussinesq approximations yielded the following expression:

$$\partial H / \partial z = -\rho c_p d\theta / dt = -\rho c_p [\partial \theta / \partial t + w(z) \partial \theta / \partial z] \quad (38)$$



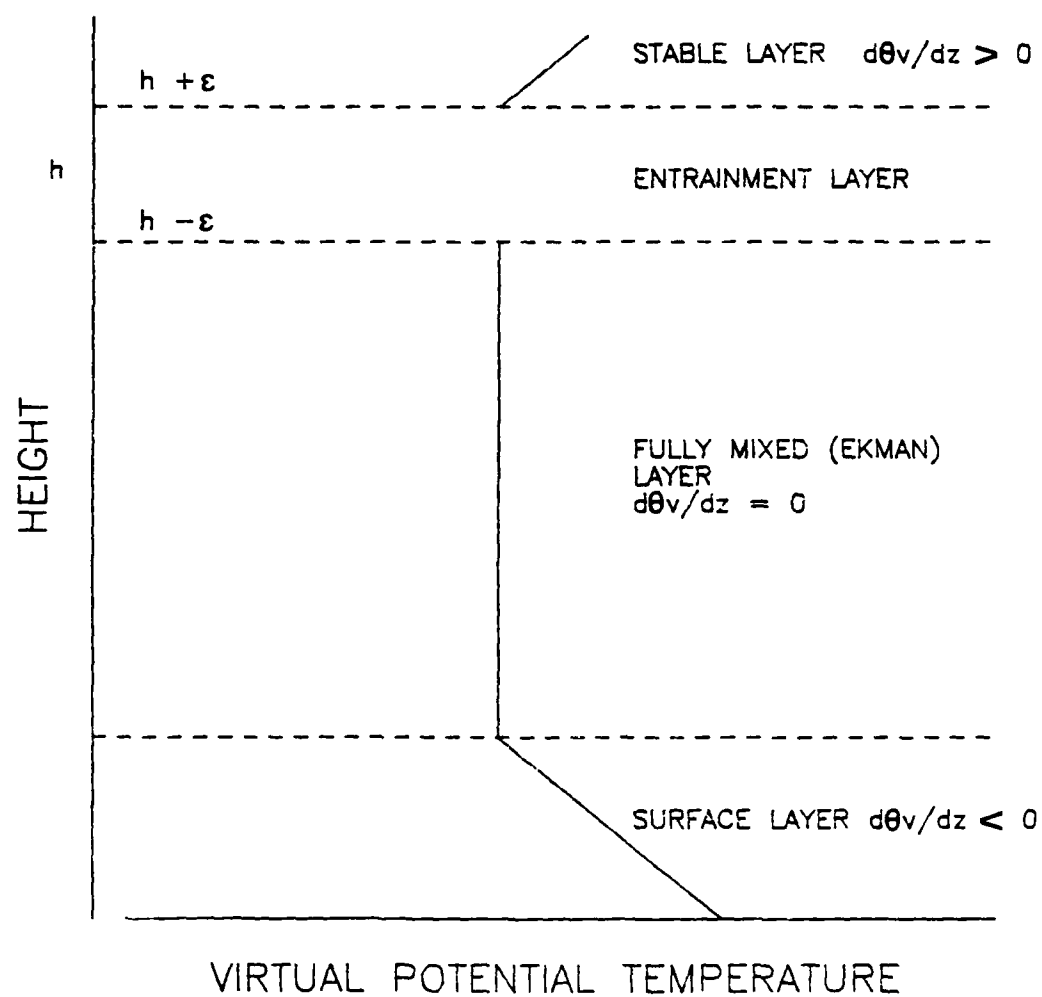


Fig. 3. Ideal atmospheric sounding of virtual potential temperature showing all layers of the planetary boundary layer.

where  $w(z)$  is the vertical velocity field and all other terms are previously defined. In the mixed layer where  $z < h$ :

$$\partial H / \partial z = -\rho c_p d\theta / dt \quad (39)$$

Implying that the sensible heat flux is linear with respect to  $z$ , Carson (1973) showed that:

$$H(z, t) = H(0, t) - z/h (H(0, t) - H(h, t)) \quad (40)$$

As  $z$  approaches  $h$ :

$$H(h, t) = H(0, t) - \rho c_p h d\theta / dt \quad (41)$$

The final step in this procedure is to take  $\theta(z, t)$  as a generalized function and integrate across the interface between the fully mixed layer and the stable layer on top of the PBL. The boundaries for the integration are from  $h - \epsilon$  to  $h + \epsilon$ , where  $\epsilon$  is some distance above and below  $h$  where the bottom and top of the entrainment layer exist. Once this is done the limit is taken as  $\epsilon \rightarrow 0$ . These procedures yield:

$$H(h, t) = -\rho c_p (dh/dt - w(h)) [\theta_s - \theta_m] \quad (42)$$

Applying equation (41) to (42) yields a form of the conservation equation presented by McNaughton and Spriggs (1986) which will be used to calculate the sensible heat flux:

$$H = \rho c_p h d\theta / dt - \rho c_p (\theta_s - \theta_m) dh/dt \quad (43)$$

Notice that the  $w(z)$  term is omitted from the final expression in equation (43). The reason for this is that the subsidence of the air

mass is not taken into account in this treatment.

An analogous conservation equation for the latent heat flux can also be determined using similar reasoning and by substituting the specific humidity ( $q$ ) for the potential temperature. The resulting equation which will be used in this approach will be:

$$LE = (\rho h dq/dt - \rho (q_{\square} - q_{\square})) \quad (44)$$

where  $L$  is the latent heat of vaporization ( $2.45 \times 10^6$  J/Kg).

In considering the height of the inversion to be used in these approaches, one must note that temperature alone is not a sufficient way to determine stability of the atmosphere. Vertical gradients in humidity also produce buoyancy effects. The presence of water vapor can be accounted for through the use of the virtual potential temperature instead of the "dry" potential temperature that appears in the equations. Thus, the inversion height of the virtual potential temperature profile was used for all calculations in this study.

## LITERATURE REVIEW

Several published studies have used the PBL similarity theory and mixed layer PBL model to estimate regional evapotranspiration and sensible heat flux, but a good verification experiment remains to be done.

PLANETARY BOUNDARY LAYER  
SIMILARITY THEORY

Saxton (1972) studied the Treynor basin watershed southwest of Omaha, Nebraska and collected daily meteorological data for the period April-October from 1969-1972. He estimated actual evapotranspiration empirically by reducing the potential evapotranspiration, found using the Penman equation, by a factor determined by plant growth and soil moisture content. The watershed in the Treynor basin was a  $0.6 \text{ km}^2$  catchment planted with corn. The model he used was largely empirical, and although it did not employ either method of interest, the data collected were used many times after.

Mawdsley and Brutsaert (1977) considered the Treynor basin data to test a planetary boundary layer similarity theory for estimating daily, three-day, weekly, and monthly regional evapotranspiration rates. They used rawinsonde data taken 30 km away at the North Omaha airport as their data. Using an expression for calculating vapor flux Brutsaert and Mawdsley (1976), they estimated the regional evapotranspiration from the upper air data. They used the similarity functions determined by Clarke and Hess (1974) in their equations. The data were analyzed using combinations of real vs. geostrophic winds and rotational vs. observed inversion heights of the PBL. The rotational inversion height

(h) was found by using the equation  $h = 0.25u_* / f$ , where  $u_*$  is the friction velocity and  $f$  is the coriolis parameter which is on the order of  $10^{-4} \text{ s}^{-1}$  at the location considered.

The above study initially assumed that the similarity function  $D$  for water vapor was equal to the function  $C$  for sensible heat. Using this assumption produced monthly evapotranspiration rates that were as much as five times greater than those reported by Saxton (1972) for the same data set. For daily estimates the discrepancy was even larger. Mawdsley and Brutsaert suggested that the assumption  $D = C$  was incorrect and the value of  $D$  was actually much smaller than  $C$ . They also concluded that the geostrophic wind should be used if the top of the PBL is determined by the rotational method and the actual measured wind if the inversion height is used. Both of these procedures give better estimates than if the height of the PBL is assumed constant as proposed by Zilitinkevich (1969). Use of the geostrophic wind resulted in 10-20% overprediction of evapotranspiration rates. This experiment had some significant problems. A design problem in the sondes used caused the hygristor to be exposed to excessive solar radiation, thus causing the sensor temperature to be higher than the actual ambient air temperature; as a result, the computed relative humidity was lower than the actual value. Mawdsley and Brutsaert suggested that these erroneous humidity values caused their results to be larger than the true value by a factor of two. Their final recommendation was to use the wind speed at the top of the boundary layer, where the height of the boundary layer as defined by the potential temperature inversion for best results. They concluded that more work was needed to properly

test the theory. One should note that all their comparisons were made against an empirical estimate of surface evapotranspiration over a small area. Even if the results agreed better, it is questionable if this would have been a proper comparison for an estimate of regional evapotranspiration due to the size of the Treynor basin and the empirical nature of the surface estimates.

Abdulmumini (1980) studied the Treynor basin as did Saxton (1972) and Mawdsley and Brutsaert (1977). He used the surface estimates of Saxton (1972) to compare his estimates for regional evapotranspiration. Using rawinsonde data and global solar radiation observations from the North Omaha Airport he calculated his estimates using two methods. First, he used a boundary layer similarity theory similar to that used as Mawdsley and Brutsaert (1977), but modified the approach by using vertically averaged specific humidities and temperatures. Second, he used the surface energy balance approach to get latent heat (LE) as the residual term.  $LE = R_n - H - S$ , where  $R_n$  is net radiation,  $H$  sensible heat flux, and  $S$  is the soil heat flux. Using this approach required estimates of net radiation and soil heat flux (assumed to be zero on a daily basis). He used an empirical relationship determined by Saxton (1972) for a grass-planted watershed in southwestern Iowa to determine net radiation. He calculated the sensible heat flux using equation (26). Vertically averaged values of potential temperature were used to account for the baroclinicity of the atmosphere. His monthly estimates of evapotranspiration agreed well with Saxton's (1972) estimates when he used the energy budget method.

In a second study, Abdulmumini applied the methods he used on the Treynor Bas'n to estimate regional evapotranspiration at the Black Vermillion watershed in northeastern Kansas. He applied the equations he calibrated on the small Treynor Basin to the larger Black vermillion watershed. He used rawinsonde data from Topeka, about 65km away. Ground measurements of evapotranspiration were not available so he used estimates of precipitation minus run-off for his calculations of ET. The agreement using this method was not very good. However, since precipitation minus run-off is not a very good estimate of evapotranspiration, one cannot accurately evaluate the validity of the estimates.

More recently, Brutsaert and Kustas (1985) conducted an experiment using PBL similarity theory and applied it to rawinsonde data taken over a hilly area in Switzerland. The average height of the hills was 95 meters. They decided to use only the soundings that met neutral conditions. Over 300 soundings were taken but only 11, considered to be close to neutral, were used. Zero plane displacement, surface roughness, and  $u_*$  were determined using the neutral wind profiles near the surface (log law). Surface evapotranspiration fluxes were estimated using the estimates of  $u_*$  and profiles of specific humidity. These flux calculations were compared to a single lysimeter and agreement was found to be excellent. This experiment was the first to compare estimated regional evapotranspiration values to a reliable ground truth measurement. However, as only one point measurement was available for comparison, one could ask the question whether this was a valid estimate of regional evapotranspiration.

# CONSERVATION EQUATION APPROACH

The mixed layer model is probably the simplest type of model used to describe the development of an inversion-capped, convective PBL (McNaughton and Spriggs, 1986). In this type of model the planetary boundary layer is assumed to be vertically well-mixed and to grow by turbulent entrainment into the relatively stable air above the inversion (Deardorff, 1983).

The virtual heat flux controls the growth rate of the PBL, which affects the rate of entrainment of drier air into the top of the layer. The introduction of the drier air influences the saturation deficit within the layer. The saturation deficit and evaporation rate are inversely related, through the surface energy budget, to the virtual heat flux (McNaughton and Spriggs, 1986).

Several early mixed layer models were developed (McNaughton, 1976 for example), but these did not take entrainment into account. De Bruin (1983) developed a model to study the behavior of the Priestly-Taylor parameter ( $\alpha$ ), which is defined as the ratio  $ET_p/ET_{eq}$  where  $ET_p$  is the potential evapotranspiration and  $ET_{eq}$  is the equilibrium evapotranspiration. Priestly and Taylor (1972) showed that in the absence of advection evaporation is directly related to the equilibrium evaporation by:

$$\lambda E = (\alpha s / (s + \gamma)) (R_n + S) \quad (45)$$

where  $\lambda$  is the latent heat of vaporization,  $E$  the evaporation,  $s$  is the slope of the saturation specific humidity-temperature curve at air temperature,  $\gamma \equiv c_p / \lambda$ ,  $R_n$  is net radiation, and  $S$  the soil heat



flux. This model used a planetary boundary layer coupled to the Penman-Monteith equation for estimating the surface fluxes of heat and water vapor. The Priestly-Taylor parameter is introduced here because the models of McNaughton (1976) and Perrier (1980) suggest that the parameter becomes unity when the air is moderately dry but is not unity when either wet or dry conditions are present. Both McNaughton's and Perrier's models assume that there is no transfer of heat or water vapor at the top of the PBL. Monteith (1981) suggested the reason for this is that they ignored entrainment into the top of the PBL. De Bruin (1983) developed a model which allowed the height of the PBL to vary and took into account the entrainment of dry air into the PBL. In his model De Bruin assumed that the flux of heat and water vapor at the surface is proportional to the fluxes of heat and water vapor at the top of the PBL. The surface layer, considered to be small, is neglected.

McNaughton and Spriggs (1986, p. 244), commenting on De Bruin's model stated,

Moisture entrainment is calculated by assuming that the downward flux of moisture at the inversion base is proportional to the evaporation rate at the ground. This closure assumption has no physical basis and is a weakness of the model.

To account for this weakness McNaughton and Spriggs (1986) replaced De Bruin's assumption of proportionality with the conservation equation for humidity (27). The model proposed by McNaughton and Spriggs (1986) follows the equations of Tennekes (1973) and Carson (1973) using the conservation equations for temperature and humidity (26-27).

To examine their ideas, McNaughton and Spriggs (1986) took data from a tower in Cabaw, the Netherlands. Using equations for transport in the lower layers and equations for entrainment into the top of the layer, they simulated the growth of the PBL ( $dh/dt$ ). After they had developed a simulated PBL they used the same initial input data to calculate their estimates of regional evapotranspiration. Although their calculated estimates agreed well with their simulation, many of the initial input figures were calculated from other data in the set, thus their simulation did not arise from independent measurements.

McNaughton and Spriggs (1986) concluded that the procedure for estimating regional evapotranspiration worked well even when the simulation of the PBL growth did not. They also concluded that the conservation equation approach showed promise for estimating regional evapotranspiration and sensible heat, even though they could not conduct a truly independent evaluation due to a lack of an independent data set.

To summarize the previous work, although several studies have been conducted, a true rigorous and independent evaluation of the ability of the PBL theories to predict regional evapotranspiration remains to be done. Brutsaert and Mawdsley (1976) and Abdulmumini (1980) did not have any actual measurements to compare their estimates against. Also, they only had two rawinsonde soundings per day, taken at sites a considerable distance away from their experimental location. Brutsaert and Kustas (1985) did have a ground truth measurement to compare their estimates against, but one can question the use of only neutral soundings as a representative sample of the real atmosphere. Also,

since they were trying to estimate regional evapotranspiration it seems questionable whether the small size of their experimental area really gave them a valid regional estimate.

The mixed-layer PBL model developed over the years by McNaughton (1976), Perrier (1980), and De Bruin (1983) had difficulties dealing with the entrainment of dry air into the top of the PBL. The approach proposed by McNaughton and Spriggs (1986) looks very promising. It is interesting that a valid estimate of regional evapotranspiration can be arrived at by solving the conservation equations for heat and water vapor budget equations without having to simulate the growth of the PBL.

## METHODS AND MATERIALS

## EXPERIMENTAL SITE

Since this research deals with estimating regional evapotranspiration, it was necessary to find a location that was large and horizontally homogeneous to fit the general assumptions. The area had to have a minimum length scale of 10 km to be considered.

The Konza Prairie Research Natural Area, located approximately 8 km south of Manhattan, Kansas was chosen as a suitable site (Fig. 4). The Konza is one of the few remaining natural prairies in the United States. It is an area of rolling hills and limestone escarpments. The entire area is vegetated by tall prairie grass and has an average altitude change of 50 meters. Although the protected area of the Konza measures only 6.6 km by 5.9 km the surrounding area, which is much larger than the required length scale, is the same type of terrain and vegetation. Considering the large-scale estimates that this research produces, the area was considered to be horizontally homogeneous and a valid site to test the theories evaluated in this paper.

After the initial site was selected, a specific site for balloon launching had to be found. The criteria for a suitable site were (1) adequate visibility to allow visual tracking of the balloons, and (2) the site had to be close to the center of the prairie. The most centrally located, nearly-level area that appeared to be at "average" elevation was finally chosen (Fig. 5).

Skies were clear throughout the experiment. Winds on 21-22 July were light at  $1-10 \text{ m s}^{-1}$ , and on 23-24 July they increased to  $>10 \text{ m s}^{-1}$ .

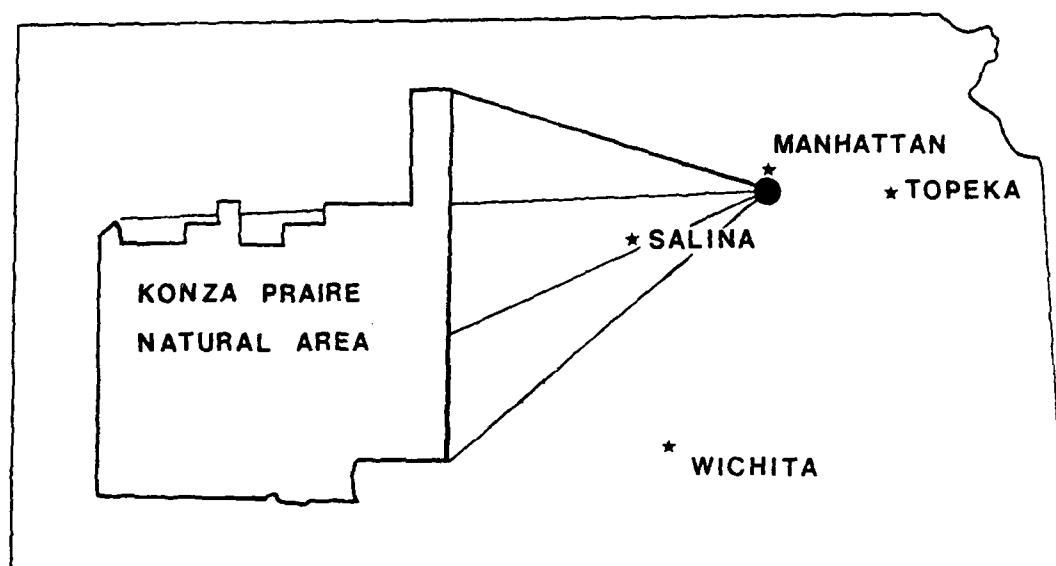


Fig. 4. Location of the Konza Prairie Research Natural Area

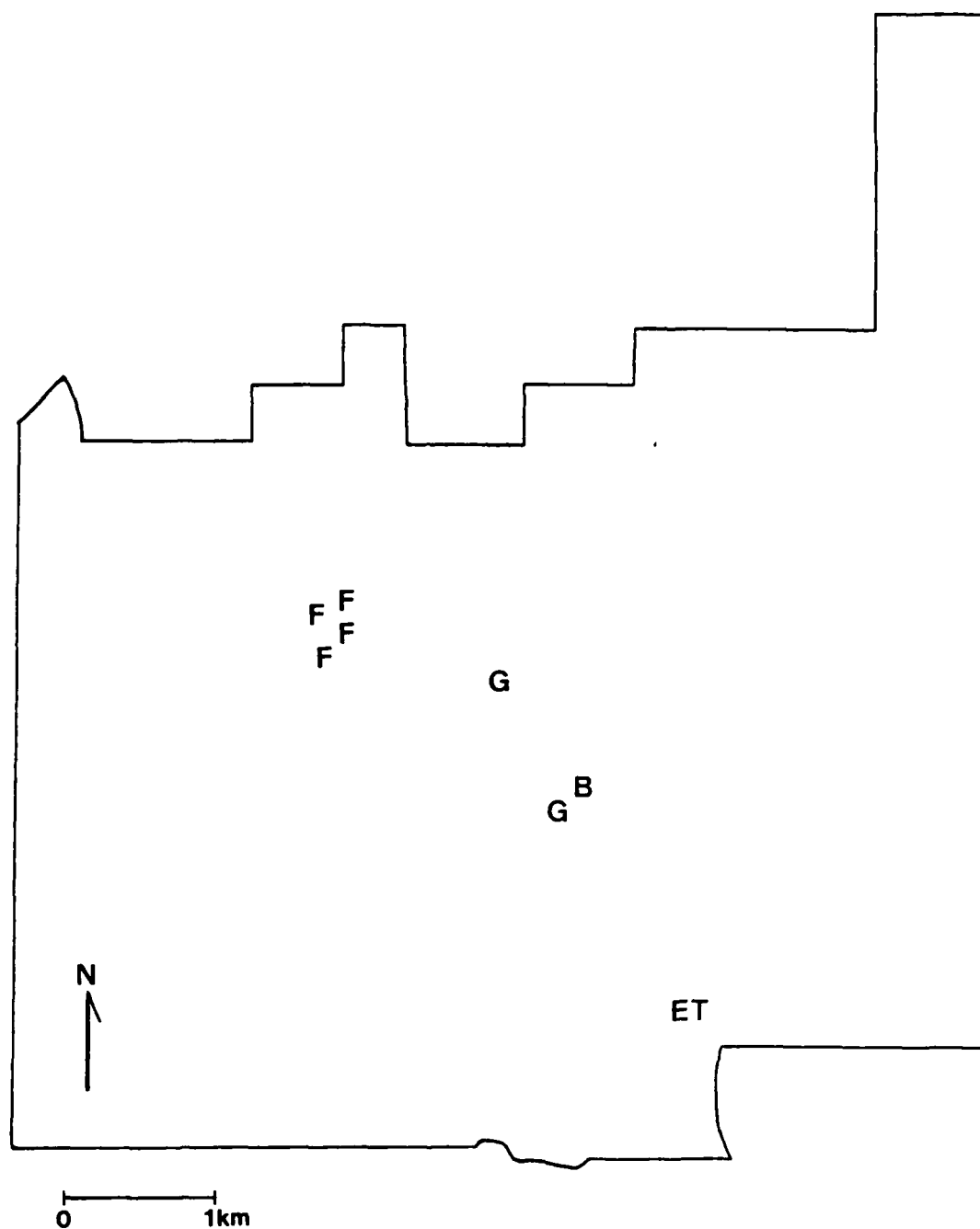


Fig. 5. Data collection locations on the Konza Prairie. B is the balloon launch site, F,G,ET are the instrument locations of Fritchen, Gay, and the Kansas State ET Lab respectively.

## DATA AQUISITION

Two types of data were required for use in this project. First, vertical soundings of temperature, humidity, and pressure were necessary as input data for both theoretical approaches. Secondly, ground truth measurements of sensible and latent heat flux were needed to verify the results of the approaches.

### Rawinsonde Data Collection

Vertical soundings of dry and wet bulb temperature and pressure were taken over a 3 1/2 day period from 21-24 July, 1986. These soundings were measured using a free-flying, 100 gm balloon with an rawinsonde package (Fig. 6) suspended from the balloon. Two types of rawinsonde packages were used. Dry bulb temperature and pressure sensors were the same for both packages, but the humidity sensors differed. The sensor most often used consisted of a wet bulb thermistor, while the other package used a carbon hygistor which measured a resistance that had to be converted into a humidity. Data were transmitted from the aisonde package to a receiver every five seconds and then recorded on magnetic tape for future processing. A back-up voice recording of the data was taken every 30 seconds during the flight of the balloon. These data were later hand-transcribed for future use. The sensitivities of the sensors on the rawinsonde package were: temperature  $\pm 0.2$  °C from +50 °C to -20 C and pressure  $\pm 3$  mb from 1000 to 300 mb.

The balloon was visually tracked using a theodolite. Readings of the azimuth and elevation were voice-recorded every 30 seconds and later transcribed for use in windspeed and direction calculations.

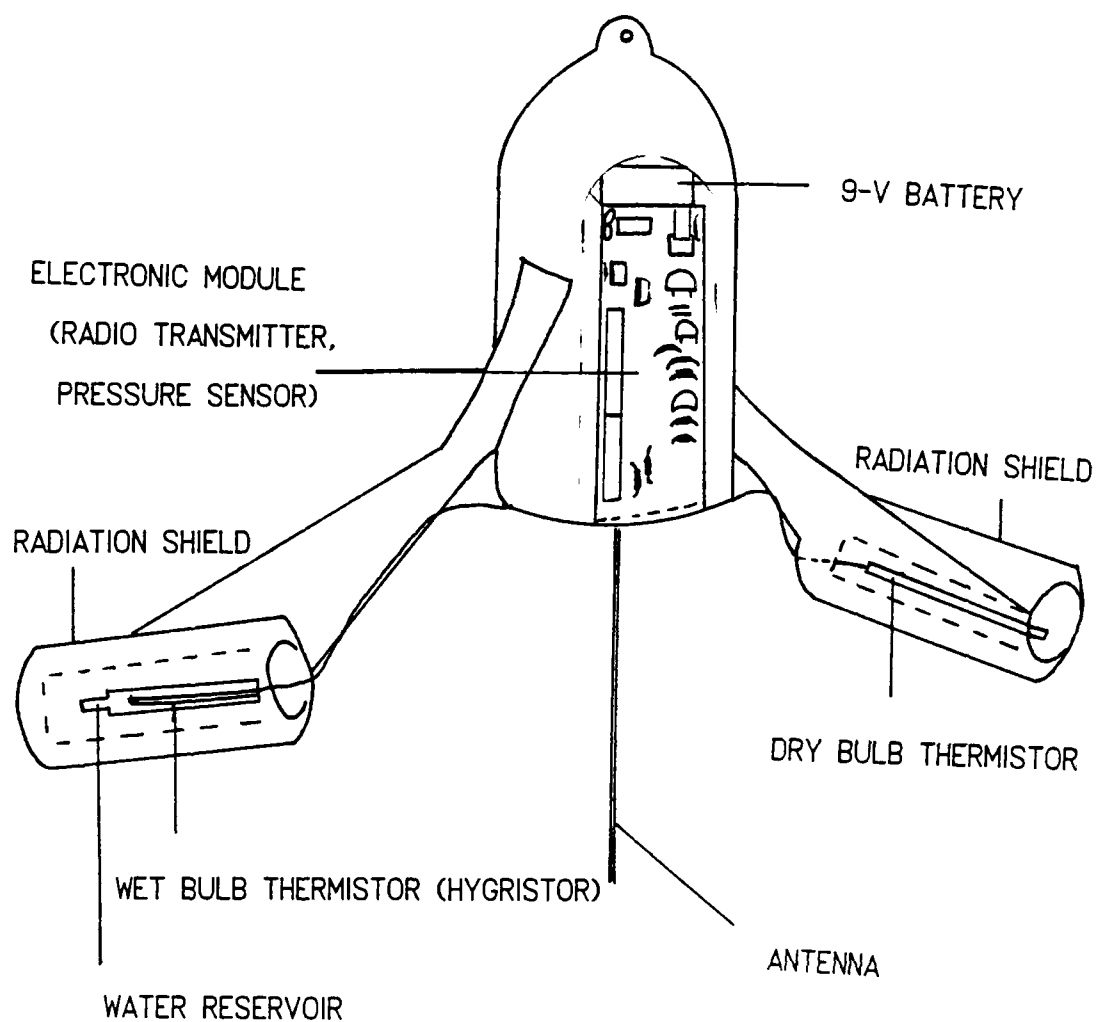


Fig. 6. Rawinsonde package



Although the average height of the planetary boundary layer was between 1-2 km, the balloon was tracked until it was out of sight and the data archived for later use.

A total of 19 soundings were taken during this period. Launch times were set up to coincide with the morning and evening sensible heat transition periods and midday evapotranspiration maximums. Using this schedule, the growth and collapse of the PBL was studied.

#### Ground Truth Data

To properly test the two theoretical approaches, it was necessary to have reliable measurements of sensible and latent heat flux at various sites around the experimental region. The comparison data for this project were supplied by three groups who were using Bowen Ratio Energy Balance stations at different sites on the prairie at the same time the soundings were taken. A total of nine sets of data were used for comparison.

Lloyd W. Gay, Professor of Watershed Management, University of Arizona, provided data from four Bowen Ratio systems. These systems were located on nearly level terrain and arrayed as two sets of two systems very close together (Fig. 5). Since these stations were situated so close together the data were averaged for each pair and are reported in this paper as if there were only two systems.

The second set of data were provided by Leo Fritchen, University of Washington. These four separate sets of data were collected on different slopes mostly in the center of the prairie (Fig. 5). The data from the slopes showed wide variability. However, since regional estimates of evapotranspiration and sensible heat were the goal, slope

data took into account the different terrain features of the prairie, making the estimates more representative of the area.

Finally, the Evapotranspiration Laboratory, Kansas State University provided one set of Bowen Ratio measurements about 1.7 km ESE of the balloon launch site (Fig. 5). Data were missing for the first day of balloon launching and for some hours on the other three days.

#### DATA HANDLING/PROCESSING

The data from the rawinsonde flights were transferred from magnetic tape to floppy disk. Problems were encountered in transferring the data from flights 10-18 of 23-24 July. A loose wire from the tape recorder to the receiver during the flight of the balloons did not allow the data to be transferred from the receiver to the magnetic tape. This resulted in only noise being transferred, and no data could be recovered. The 30-second, manually recorded data were used for these soundings instead of the automatically recorded data. Although the resolution and, therefore, quality of the manually recorded soundings are not as good as the ones from the automatically recorded soundings, they are usable to make estimates for most of the flights.

Dry and wet bulb temperatures were used with the Goff-Gratch formula (List, 1971) to obtain saturation vapor pressures. Actual vapor pressures were calculated using the psychrometric equation:

$$e_a = e_s(T_w) - [(P c_p)/(L \epsilon)] (T - T_w) \quad (46)$$

where  $e_a$  is the actual vapor pressure (mb),  $e_s$  the saturation vapor pressure (mb) at wet bulb temperature,  $P$  is the pressure in mb, and  $\epsilon$  is the ratio of the mass of water to the mass of air (.622).

Once the actual vapor pressure was calculated the specific humidity (q) was found using:

$$q = \epsilon e_a / (P - .378 e_a) \quad (47)$$

Specific humidity was then used to calculate virtual temperature ( $T_v$ ), potential temperature ( $\theta$ ), and virtual potential temperature ( $\theta_v$ ).

To determine the heights at which the rawinsonde was collecting the data, the integrated form of the hydrostatic equation:

$$\Delta Z = [(R_d T_{avg})/g] [\ln(P_1) - \ln(P_2)] \quad (48)$$

was used where  $\Delta Z$  is the change in height between two pressure levels,  $g$  is the acceleration due to gravity,  $R_d$  is the gas constant for dry air,  $T_{avg}$  the average virtual temperature between two readings, and  $P_1$  and  $P_2$  the pressures at the lower and upper layer being considered.

Once all the automatically recorded data were processed the manually recorded 30-second data were processed to yield windspeed and direction vs. height. First, the elevation angle and azimuth were converted to radians. The radial distance was then calculated by dividing the height by the tangent of the elevation angle. Next, the components were put into cartesian coordinates by multiplying the radial distance by the sine of the azimuth and the cosine of the azimuth to get  $x$  and  $y$ , respectively. The average windspeeds in the  $u$  and  $v$  direction over the time interval between theodolite readings, in this case 30 seconds, were calculated. Finally, the wind direction was found by taking the arc tangent of  $u/v$  and converting to degrees.

In the cases of sondes 10-18, the 30-second data also provided the other variables that were calculated for the automatically recorded data.

#### Determination of Inversion Height

Determination of the virtual potential temperature inversion height was the most critical operation for both approaches. Soundings of virtual potential temperature were plotted, and the top and bottom of the entrainment layer (Fig. 3.) were determined by visual inspection. This process was clear-cut in the soundings plotted using automatically recorded data.

The bottom of the entrainment layer was determined by the point in the sounding where the virtual potential temperature deviated from an isothermal pattern defines the top of the mixed layer. The top of the entrainment layer was chosen as the point where the virtual potential temperature began a monotonic increase, as would be expected in the stable layer above the PBL. This point was easily distinguished in all soundings using the automatically recorded data. The true inversion height lay somewhere in this entrainment layer.

After close analysis of the soundings it was determined that in all the soundings there was a point within each entrainment layer where the virtual potential temperature increased over at least three consecutive points. The lower of these points was chosen as the inversion height. Samples of the virtual potential temperature and specific humidity plotted versus height above ground level for one full day (July 22) are shown in Figs. 7-16.

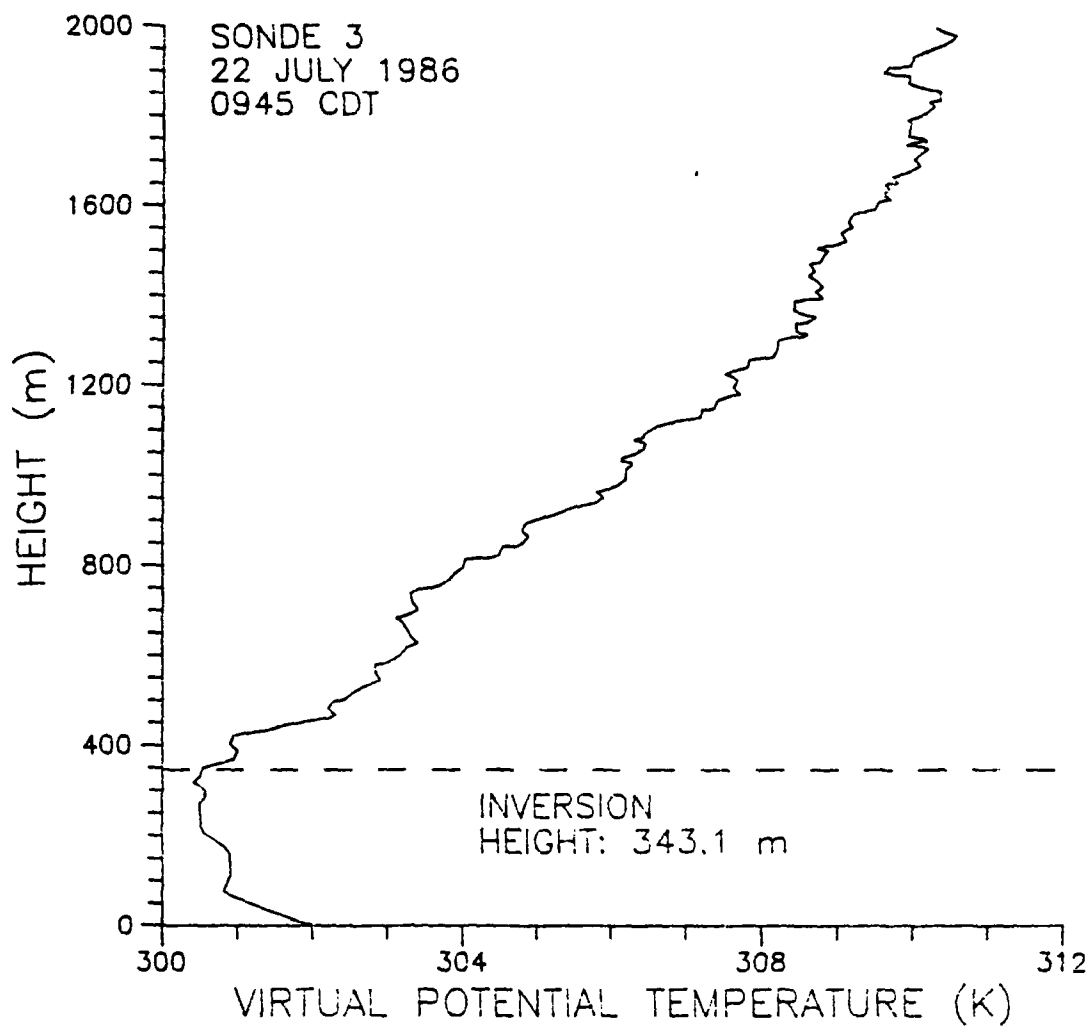


Fig. 7. Virtual potential temperature sounding for July 22, 1986, 0945 Central Daylight Time

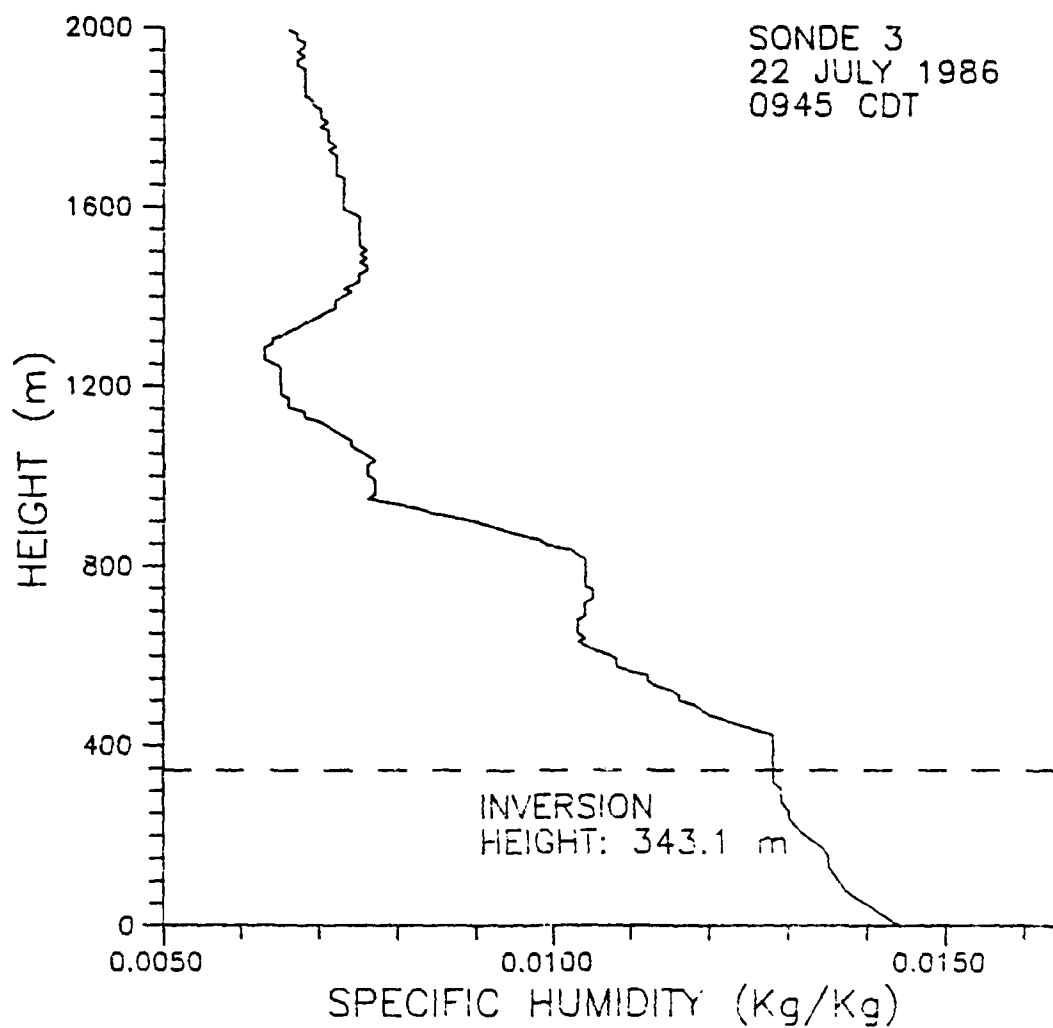


Fig 8. Specific humidity sounding for July 22, 1986, 0945 Central Daylight Time

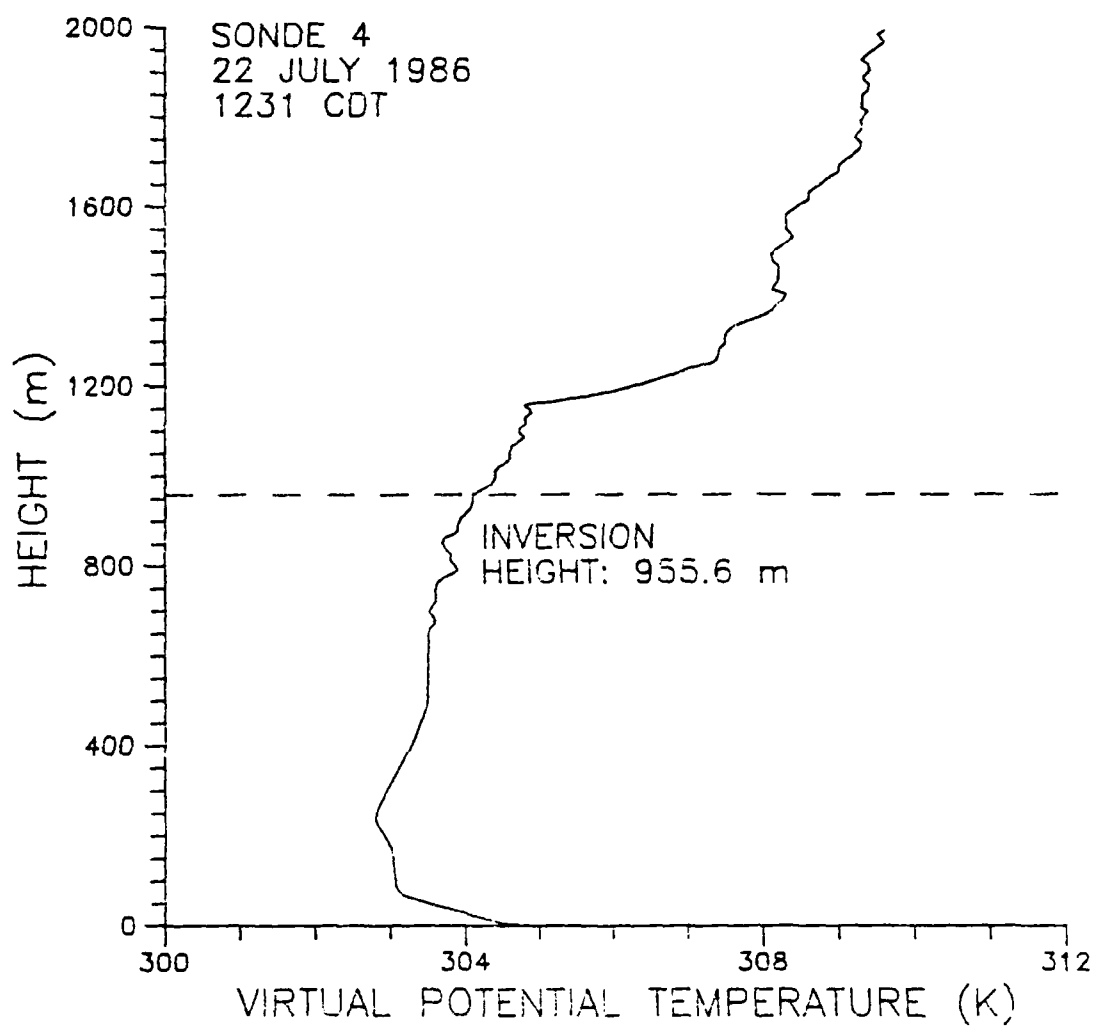


Fig. 9. Virtual potential temperature sounding for July 22, 1986, 1231 Central Daylight Time

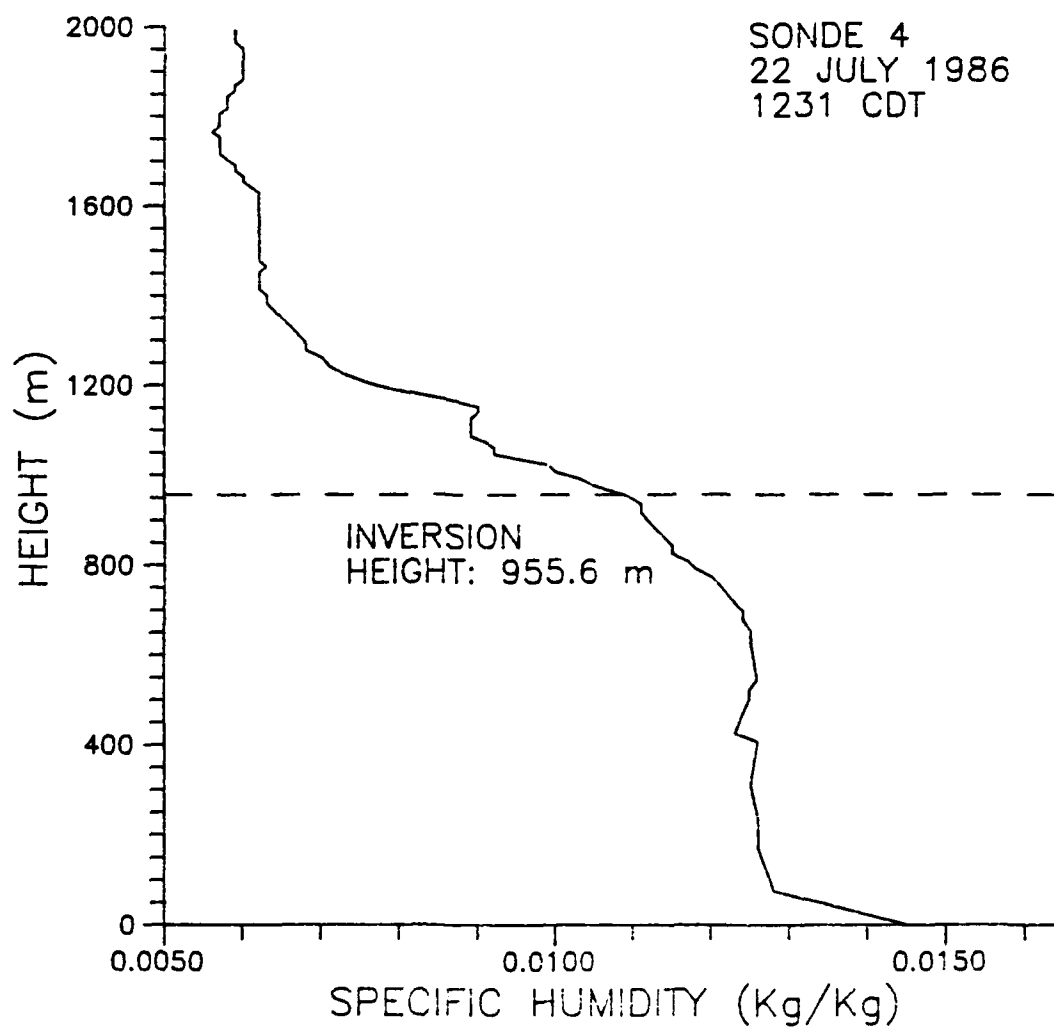


Fig. 10. Specific humidity sounding for July 22, 1986, 1231, Central Daylight Time



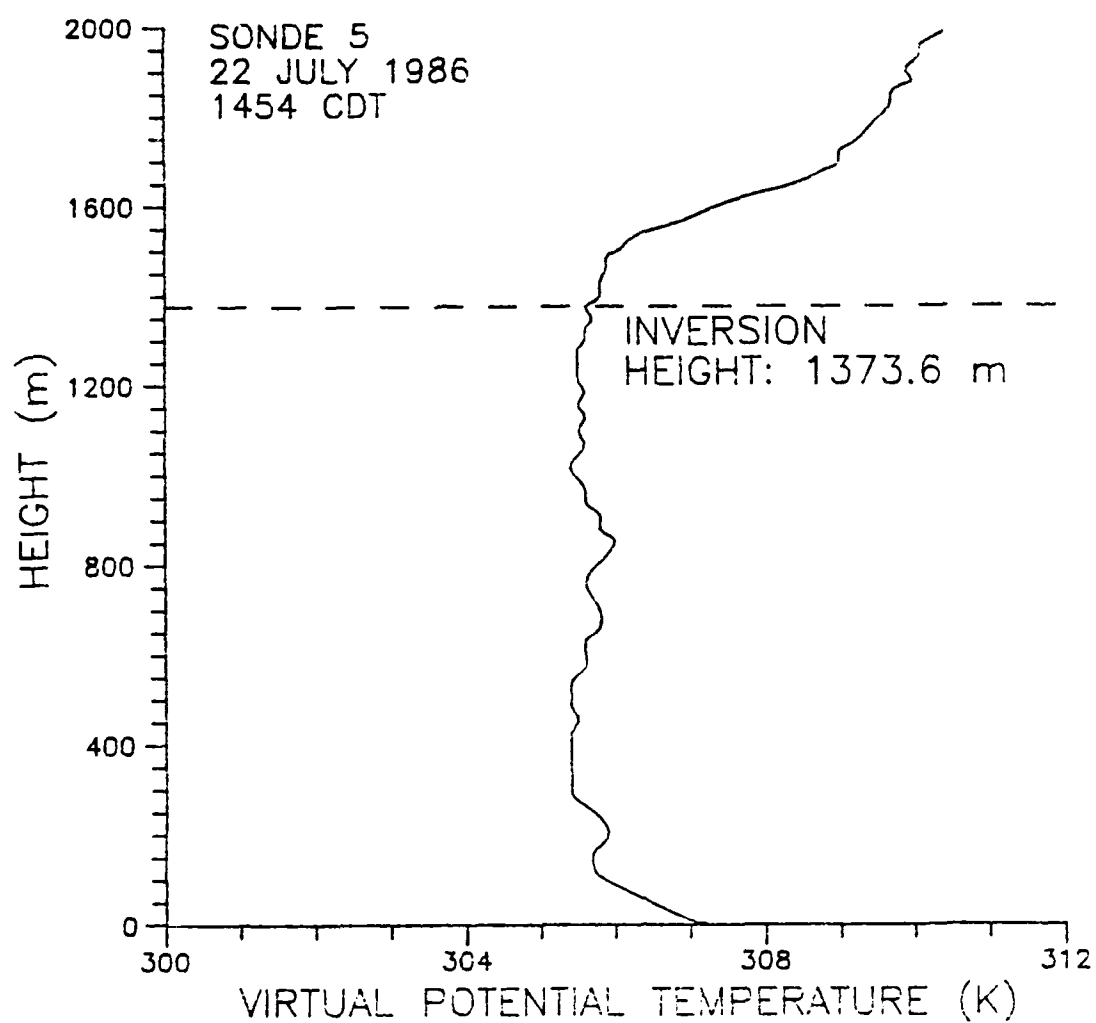


Fig. 11. Virtual potential temperature sounding for July 22, 1986,  
1454 Central Daylight Time

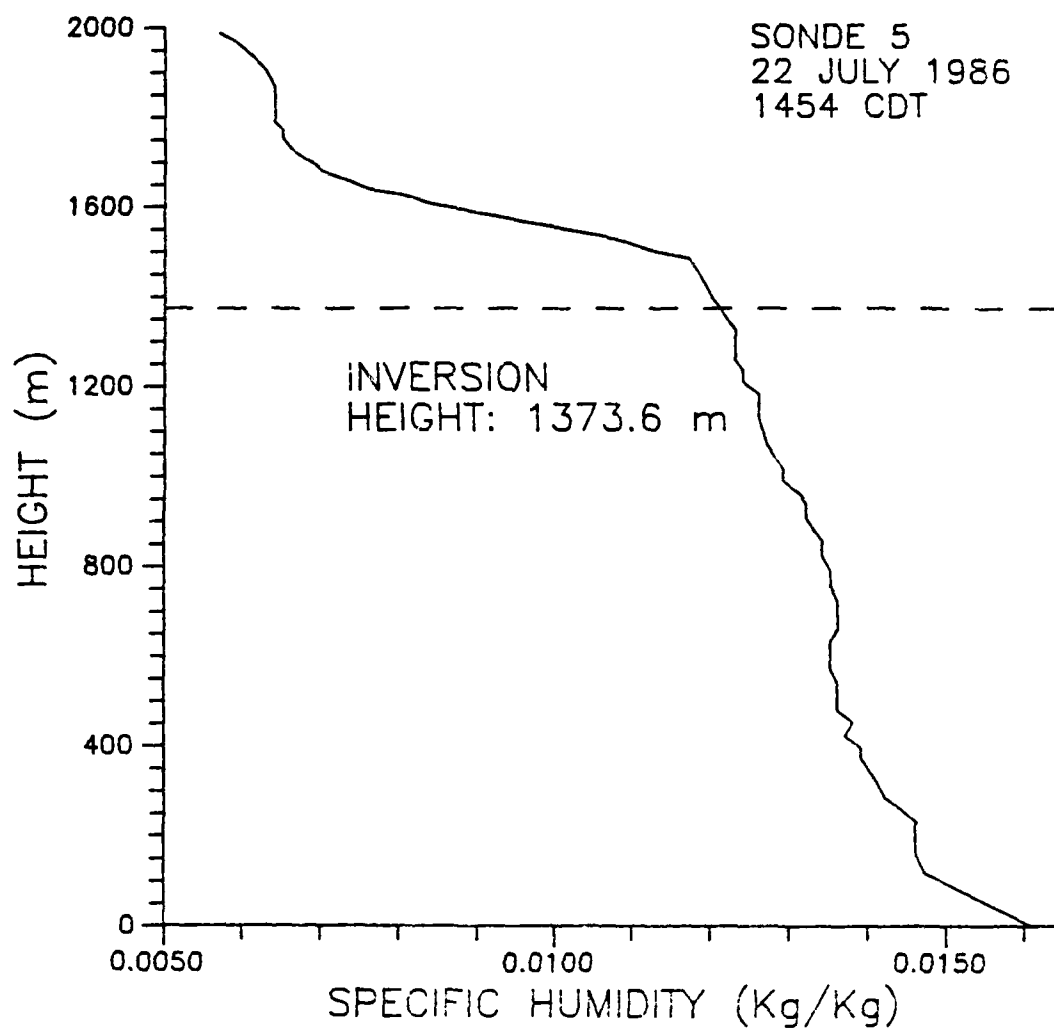


Fig. 12. Specific humidity sounding for July 22, 1986, 1454, Central Daylight Time

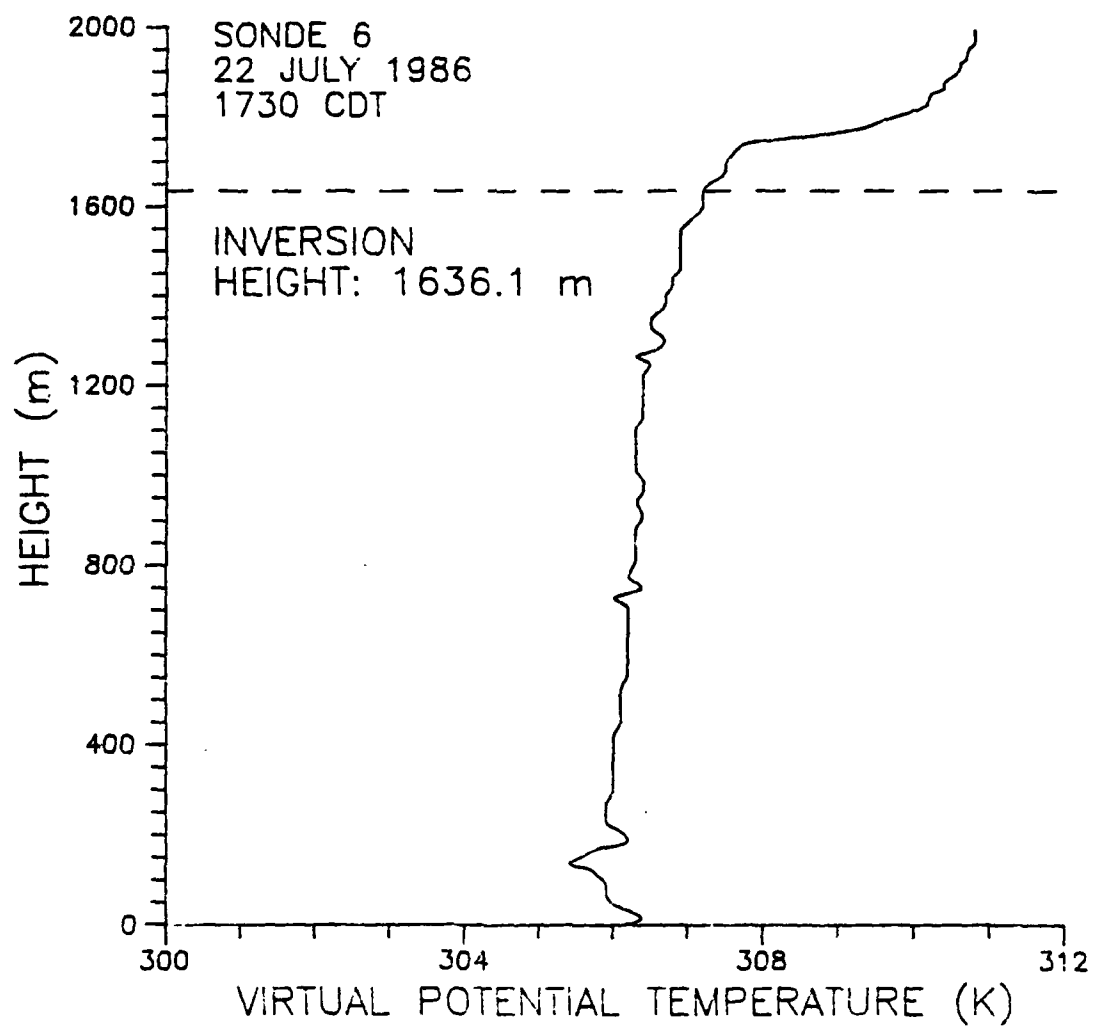


Fig. 13. Virtual potential temperature sounding for July 22, 1986,  
1730 Central Daylight Time

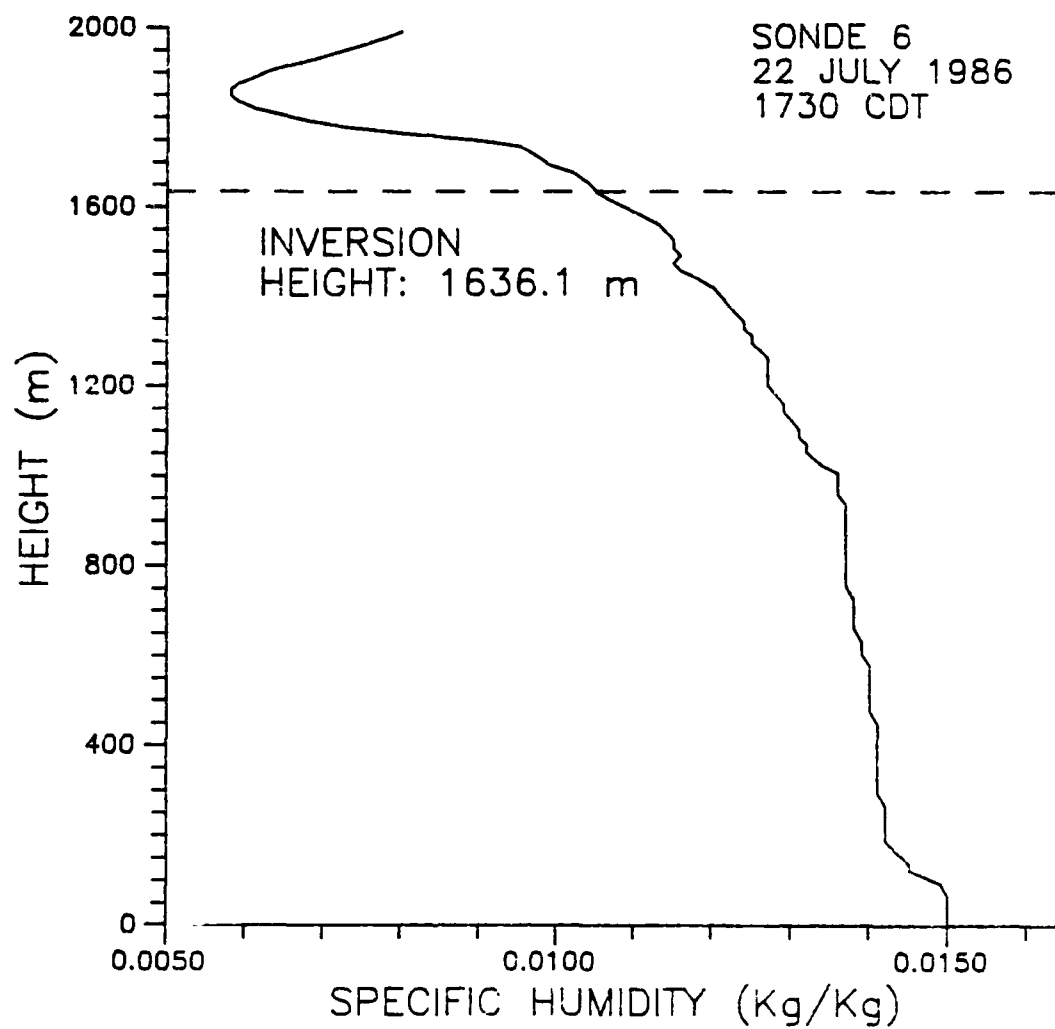


Fig. 14. Specific humidity sounding for July 22, 1986, 1730. Central Daylight Time

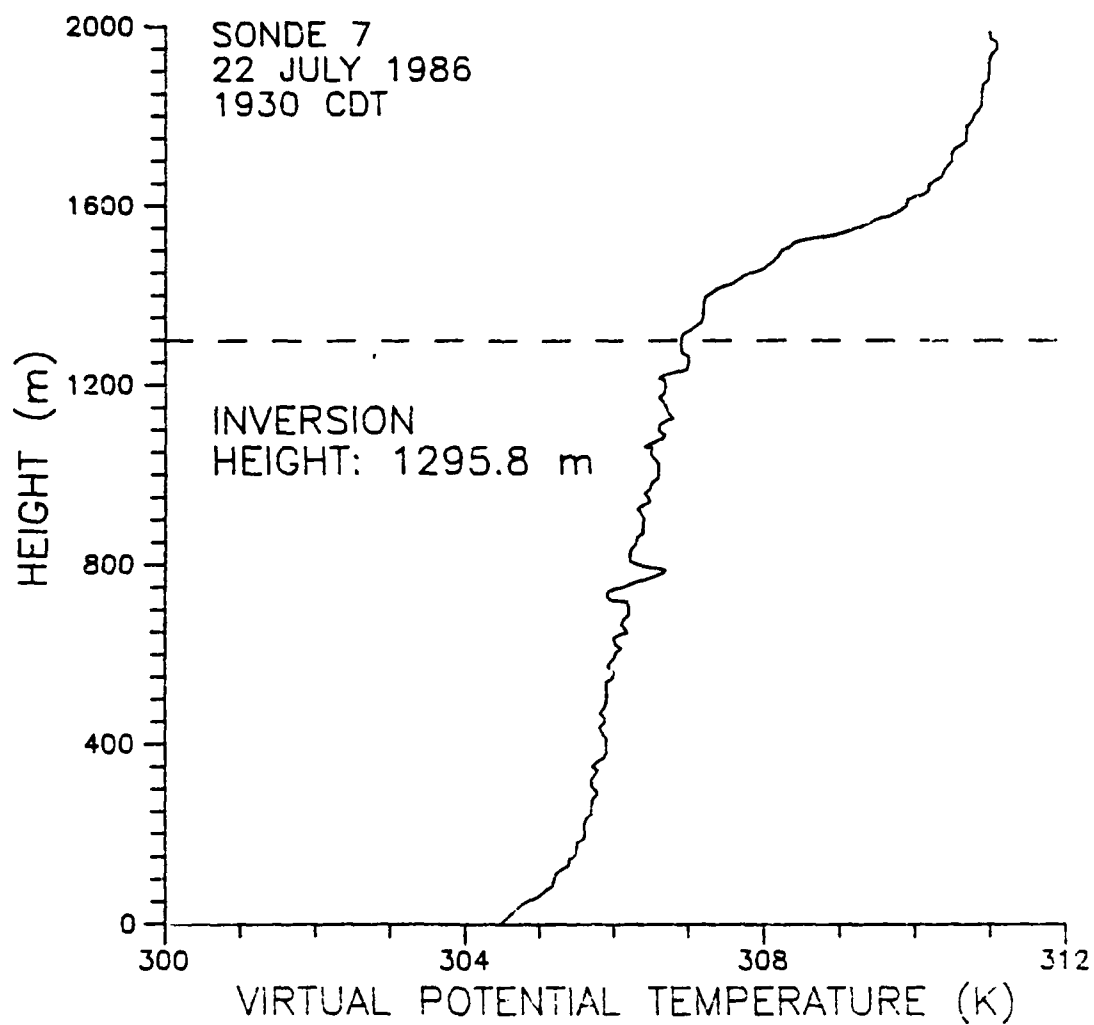


Fig. 15. Virtual potential temperature sounding for July 22, 1986, 1930 central Daylight Time

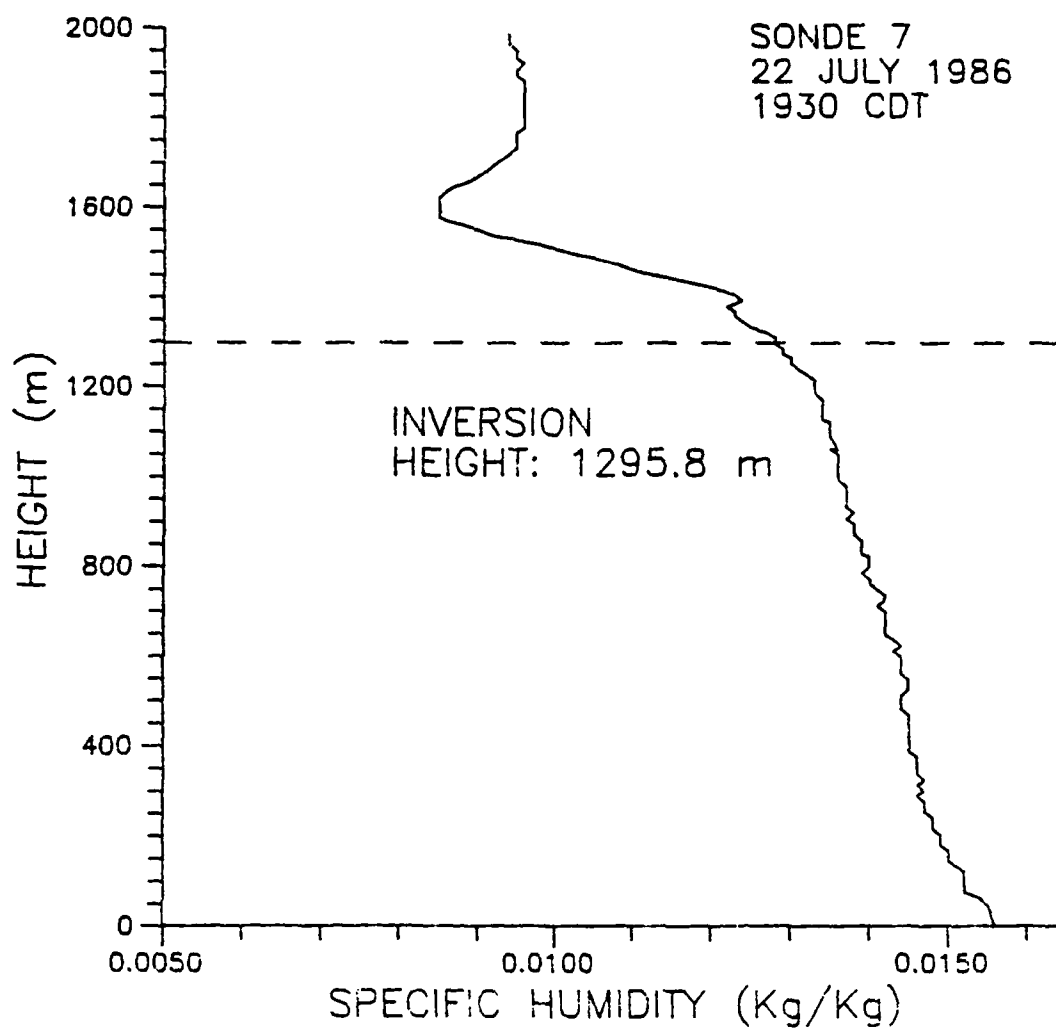


Fig. 16. Specific humidity sounding for July 22, 1986, 1930, Central Daylight Time

For the soundings plotted using manually recorded 30-second data, a less accurate visual determination of the inversion height was used. Since the points on these soundings were so far apart, much of the data in the entrainment layer was lost. Thus, the inversion height was determined at the point where the virtual potential temperature showed a distinctive monotonically increasing trend. This process yielded inversion heights that were somewhat higher than they should have been. Admittedly this procedure is less rigorous, but under the circumstances it was the best available.

After examining the data, it is clear that a slight error in choosing the inversion height using the automatically recorded soundings would yield a smaller error than the manually recorded soundings.

The specific humidity soundings show dry air in the stable layer above the PBL. If the inversion height selected is too high the dry air will cause a larger gradient in the specific humidity than is really present. This gradient will cause both approaches to yield much higher evapotranspiration rates. Since the manually recorded data are subject to a greater possible error in height determination, possible error in ET values is greater than for the other soundings.

#### Planetary Boundary Layer Similarity Theory

The first step was to find the Monin-Obukov length ( $L$ ) which is required to determine the stability functions. Since calculation of  $L$  requires prior knowledge of the sensible and latent heat fluxes, it was necessary to determine it using an iterative procedure. The bulk Richardson number was determined using the processed rawinsonde data

and equation (35). The bulk Richardson number is a function of the  $\theta_v$  at the surface, which cannot be determined directly from the rawinsonde data. Therefore,  $\theta_v$  and other surface variables ( $T_o$ ,  $\theta_o$ , and  $q_o$ ) were determined using a logarithmic interpolation procedure.

Using the surface layer similarity relation given by Businger et al. (1971):

$$(\bar{\theta} - \bar{\theta}_o)/T_* = 1/k [\ln z/z_o - \psi_2] \quad (49)$$

where  $T_*$  is a temperature scale ( $= H/ku_*\rho c_p$ ) and  $\psi_2$  is a dimensionless height which is a function of  $z/L$ , any surface variable can be determined from a simultaneous equation formed from the first two rawinsonde levels. The final equation is:

$$x_o = x_1 - (x_2 - x_1) \left[ \frac{[\ln(z_1/z_o) - \psi_2(z_2/L)]}{[\ln(z_2/z_1) - \psi_2(z_1/L)]} \right] \quad (50)$$

where  $x_o$  is an interpolated value of the variable to  $z = z_o$ , subscripts 1 and 2 are first and second rawinsonde levels, respectively, and  $\psi_2(z/L)$  is equal to  $\psi_2$  evaluated at  $(z/L)$ .  $z_o$  values used throughout this research are 10cm for momentum determined by using empirical relations with plant height (Stanhill, 1965) and 3cm for calculations involving values of heat and humidity. Due to the type of vegetation that was on the prairie, zero plane displacement was so small that it was ignored in the calculations.

$\psi_2$  was determined for both rawinsonde levels using these equations (Businger et al., 1971):



$$\psi_2 = \ln [((1 + (1 - 9z/L))^{1/2})/2] \quad \text{for } z/L < 0 \quad (51)$$

$$\psi_2 = 6.27 z/L \quad \text{for } z/L > 0 \quad (52)$$

Note that  $\psi_2$  is derived from (49) for the temperature profile only. By using the assumption that  $\phi_h = \phi_w$  (Pruitt et. al., 1973), where  $\phi_h$  and  $\phi_w$  are the universal  $\phi$ -functions for heat and water vapor, it can be implied that  $\psi_2$  is the same for both temperature and humidity profiles.

The value for  $\theta_{v_o}$  was determined using a two step-iterative process. First, an estimate of the bulk Richardson number was obtained using  $\theta_v$  at the first level (1.5 m). A first estimate of  $L$  was then determined using the first estimate of the bulk Richardson number. The first estimate of  $L$  was then applied to equation (50) to yield first estimates of  $T_o$ ,  $q_o$ , and  $\theta_o$ . The procedure was then repeated for a second estimate of  $L$ ,  $T_o$ ,  $q_o$ , and  $\theta_o$ . Further iterations showed no difference in the values, so the process was stopped after two iterations.

Once  $L$  was determined the similarity parameters  $A, B,$  and  $C$ , were determined using the expressions presented by Yamada (1976). In addition,  $D$  was assumed to be  $0.65^\circ\text{C}$ . After the parameters were found it was a simple matter of applying equations (32-34) to get estimates of the surface fluxes of sensible and latent heat. Table 1 lists the calculated values for the bulk Richardson Numbers, Monin Obukhov length, and interpolated surface values for each sounding.

Two separate approaches were taken when applying equations (32-34). Values at the top of the PBL for potential temperature, virtual temperature, virtual potential temperature, and specific humidity were used in the first approach. Second, values for the same parameters

Table 1. Calculated values of Bulk Richardson Number ( $Ri_b$ ), Monin-Obukov length ( $L$ ) (m), friction velocity ( $u_*$ ) (m/s), and interpolated values of temperature ( $^{\circ}K$ ), potential temperature ( $^{\circ}K$ ), virtual potential temperature ( $^{\circ}K$ ), and specific humidity (Kg/Kg) at the surface.

| SONDE | $Ri_b$ | $L$    | $u_*$ | $T_o$ | $\theta_o$ | $\theta_{v_o}$ | $q_o$ |
|-------|--------|--------|-------|-------|------------|----------------|-------|
| 1     | -1.82  | -50.9  | .28   | 300.6 | 303.8      | 306.6          | .0144 |
| 2     | .91    | 67.4   | .13   | 299.3 | 302.6      | 305.2          | .0133 |
| 3     | -1.60  | -20.3  | .31   | 300.0 | 302.8      | 306.0          | .0149 |
| 4     | -7.82  | -9.5   | .25   | 302.8 | 305.7      | 309.1          | .0155 |
| 5     | -6.78  | -15.4  | .32   | 305.1 | 308.0      | 311.9          | .0169 |
| 6     | -1.63  | -79.6  | .32   | 303.5 | 306.5      | 309.8          | .0151 |
| 7     | .80    | 104.9  | .24   | 301.0 | 304.4      | 307.4          | .0156 |
| 8     | -.06   | -459.5 | .61   | 299.6 | 302.6      | 306.0          | .0179 |
| 9     | -.87   | -85.4  | .56   | 306.2 | 309.6      | 313.2          | .0178 |
| 10    | -2.04  | -41.9  | .54   | 310.0 | 312.6      | 316.9          | .0178 |
| 11    | -.51   | -225.6 | .57   | 306.1 | 309.0      | 313.4          | .0181 |
| 13    | -.09   | -341.4 | .54   | 301.9 | 304.1      | 307.9          | .0152 |
| 14    | -.23   | -222.8 | .81   | 305.9 | 309.4      | 313.6          | .0185 |
| 15    | -.28   | -196.4 | .80   | 308.3 | 311.7      | 316.1          | .0180 |

were averaged over the entire planetary boundary layer and then applied to equations (32-34). This second approach was reported by Abdulmumini (1980) to yield better results.

#### Conservation Equation Approach

To solve the conservation equations (43 and 44) for temperature and humidity requires determination of the rate of change of potential temperature, specific humidity, and height with respect to time. A cubic spline routine was used to determine the derivatives of these variables at any point in time throughout each day. The spline routine calculates the values for the coefficients that fit the polynomial  $S(x) = A + Bx + Cx^2 + Dx^3$ . To get the rate of change the polynomial is differentiated. These values were then put into equations (43 and 44) to determine the surface fluxes of sensible and latent heat.

Finite differencing was also applied and found to have similar results. The cubic spline routine was selected because it gave one more point than finite differencing.

## RESULTS AND DISCUSSION

Rawinsonde measurements of temperature, humidity, and pressure made over the period of July 21-24, 1986 were analyzed to determine the sensible and latent heat fluxes. The data of July 21-22 were collected under light wind conditions (1-5 m/s) while the data of July 23-24 were collected under strong wind conditions of  $> 10$  m/s. The height of the planetary boundary layer over this period ranged from 300 meters in the morning up to 1650 meters when fully developed in the late afternoon.

Planetary Boundary Layer Similarity Theory and the conservation equations proposed in the McNaughton and Spriggs (1986) Mixed Layer PBL Model were used to estimate the regional surface fluxes of sensible and latent heat.

A total of 19 soundings were taken over the 3 1/2 day period but only 14 were usable. Automatically recorded data were available for soundings 1-9. Soundings 10-19 utilized only 30-second readings taken as a system back-up. The quality of these soundings was poor compared to soundings 1-9, but reasonable estimates were made from most of them. Sounding 12 was unusable due to erroneous humidity data in the lower 100m of the sounding making it impossible to calculate realistic surface values of temperature and humidity. Also, many of the data points were missing, thus an inversion could not be detected. Soundings 16-19 could not be used due to missing data between the 30-second readings, making proper identification of the inversion height impossible.

A sensitivity analysis was done to determine how sensitive the approaches were to the inversion height. This analysis was conducted because the inversion height selected not only changes the height but also affects the humidity and temperature gradients and the wind at the top of the PBL.

To test the sensitivity of the method used in this research to determine inversion heights, flux values were calculated for the layers directly above and below the selected inversion height. Results of this test show that for sensible heat there is an average of  $\pm 10.6\%$  difference in the flux and a  $\pm 9.6\%$  difference for latent heat flux.

A second test was done to find the difference between the selected inversion height and the height at the top of the entrainment layer. This height would be the obvious choice for the inversion height if the entrainment layer were not taken into account. This test yielded a 32.3% difference for sensible heat and a 19.2% difference for latent heat.

These tests were conducted using only the soundings plotted using the 5-second data. Testing the 30-second sounding would lead to gross differences, due to the large change in the humidity and temperature gradients created when a height above the true top of the PBL is selected.

A discussion of the results from the PBL Similarity Theory and Conservation Equation Approach follows. Tables 2 and 3 show estimates from these approaches compared against averaged data measured by nine Bowen Ratio systems.

Table 2. Sensible heat (H) estimates ( $W\ m^{-2}$ ) from all approaches compared against ground truth data.

|         | AVERAGE<br>MEASURED VALUES<br>ALL GROUND<br>STATIONS | STANDARD<br>DEVIATION | CONSERVATION<br>EQUATION<br>APPROACH | PBL SIMILARITY<br>THEORY<br>AVERAGED<br>VALUES | TOP OF PBL<br>VALUES |
|---------|--|-----------------------|--------------------------------------|--|----------------------|
|         | H  | H                     | H                                    | H  | H                    |
| JULY 21 |  |                       |                                      |  |                      |
| 1530    | 176.6  | 33.6                  | 39.8                                 | 17.3   | 15.3                 |
| 1900    | 18.5   | 24.3                  | 32.3                                 | -5.2   | -6.3                 |
| JULY 22 |  |                       |                                      |  |                      |
| 1000    | 161.5  | 41.5                  | 119.3                                | 90.5   | 100.0                |
| 1230    | 240.3  | 62.9                  | 257.7                                | 88.0   | 66.7                 |
| 1500    | 206.1  | 54.7                  | 242.0                                | 111.8  | 113.0                |
| 1730    | 68.6   | 27.1                  | -0.5                                 | 3.6  | -22.8                |
| 1930    | -21.1  | 17.7                  | -1.5                                 | -16.8  | -24.3                |
| JULY 23 |  |                       |                                      |  |                      |
| 0930    | 131.4  | 51.9                  | 227.3                                | 10.0   | 20.3                 |
| 1130    | 204.2  | 48.0                  | 405.5                                | 140.5  | 152.5                |
| 1400    | 213.8  | 91.0                  | 94.1                                 | 190.8  | 218.7                |
| 1600    | 134.0  | 50.4                  | 111.9                                | -15.9  | -29.8                |
| JULY 24 |  |                       |                                      |  |                      |
| 0830    | 19.4   | 20.9                  | 181.8                                | -43.5  | -89.4                |
| 1030    | 122.4  | 51.5                  | 202.5                                | 51.8   | -3.2                 |
| 1200    | 182.3  | 75.2                  | 236.3                                | 72.6   | 57.3                 |

NOTE: TIME IS CENTRAL DAYLIGHT TIME

Table 3. Evapotranspiration estimates from all approaches compared against ground truth data.

|         | AVERAGE<br>MEASURED VALUES<br>ALL GROUND<br>STATONS |                     | STANDARD<br>DEVIATION | CONSERVATION<br>EQUATION<br>APPROACH |                     | PBL SIMILARITY<br>THEORY |                     |                      |                     |
|---------|---|---------------------|-----------------------|--------------------------------------|---------------------|--------------------------|---------------------|----------------------|---------------------|
|         |   |                     |                       |                                      |                     | AVERAGED<br>VALUES       |                     | TOP OF PBL<br>VALUES |                     |
|         | ET  |                     |                       | ET                                   |                     | ET                       |                     | ET                   |                     |
|         | W m <sup>-2</sup>                                   | mm/hr <sup>-1</sup> |                       | W m <sup>-2</sup>                    | mm/hr <sup>-1</sup> | W m <sup>-2</sup>        | mm/hr <sup>-1</sup> | W m <sup>-2</sup>    | mm/hr <sup>-1</sup> |
| JULY 21 |   |                     |                       |                                      |                     |                          |                     |                      |                     |
| 1530    | 278.1   | .41                 | 76.8                  | 94.0                                 | .19                 | 122.3                    | .18                 | 164.4                | .24                 |
| 1900    | 115.6   | .17                 | 50.3                  | 83.3                                 | .12                 | 13.9                     | .02                 | 23.5                 | .03                 |
| JULY 22 |   |                     |                       |                                      |                     |                          |                     |                      |                     |
| 1000    | 198.9   | .29                 | 34.1                  | 173.5                                | .25                 | 129.8                    | .19                 | 162.7                | .29                 |
| 1230    | 353.8   | .52                 | 76.3                  | 338.6                                | .50                 | 217.8                    | .32                 | 294.9                | .43                 |
| 1500    | 362.6   | .53                 | 88.8                  | 358.8                                | .53                 | 260.2                    | .38                 | 359.6                | .53                 |
| 1730    | 239.2   | .35                 | 65.4                  | 265.8                                | .39                 | 109.5                    | .16                 | 266.4                | .39                 |
| 1930    | 64.4  | .12                 | 36.2                  | 182.3                                | .27                 | 39.7                     | .06                 | 75.2                 | .11                 |
| JULY 23 |   |                     |                       |                                      |                     |                          |                     |                      |                     |
| 0930    | 158.4   | .23                 | 19.1                  | 202.4                                | .30                 | 233.5                    | .34                 | 258.2                | .38                 |
| 1130    | 334.3   | .49                 | 56.6                  | 581.8                                | .85                 | 388.1                    | .57                 | 399.0                | .59                 |
| 1400    | 382.9   | .56                 | 116.9                 | 252.4                                | .37                 | 482.1                    | .71                 | 582.6                | .85                 |
| 1600    | 332.9   | .49                 | 87.7                  | 353.8                                | .52                 | 270.4                    | .40                 | 377.2                | .55                 |
| JULY 24 |   |                     |                       |                                      |                     |                          |                     |                      |                     |
| 0830    | 140.4   | .21                 | 30.0                  | 56.8                                 | .08                 | 146.7                    | .2                  | 255.8                | .38                 |
| 1030    | 309.1   | .45                 | 110.6                 | 221.2                                | .33                 | 494.5                    | .73                 | 654.1                | .96                 |
| 1200    | 393.1   | .58                 | 79.5                  | 233.9                                | .34                 | 271.8                    | .40                 | 377.1                | .55                 |

NOTE: TIME IS CENTRAL DAYLIGHT TIME

## PLANETARY BOUNDARY LAYER SIMILARITY THEORY

The results in Tables 2 and 3 indicate that of the two approaches used in this study, using the temperature and humidity values at the top of the PBL gave better results than using the vertically averaged values of temperature and humidity. This is contrary to the results of Abdulmumini (1980), who got better results using vertically averaged values.

### Sensible Heat Flux

Estimates of sensible heat flux calculated using the PBL Similarity Theory were not very reasonable. Table 2 shows that both methods yielded estimates within one standard deviation only three out of fourteen times.

Figs. 17-20 show the comparisons for July 22 and 23 of the calculated estimates using both approaches versus the ground truth data for sensible heat flux. These figures illustrate the better agreement obtained when using the values at the top of the PBL. Figs. 17-20 show that the similarity sensible heat flux estimates were consistently low, except in one case for July 23.

Figs. 21 and 22 present the results of both approaches of this theory against a 1:1 line. Estimates obtained using values at the top of the PBL show a slightly better relationship than those calculated using vertically averaged values. The underestimated trend of the sensible heat flux values is clearly seen in these 1:1 plots.



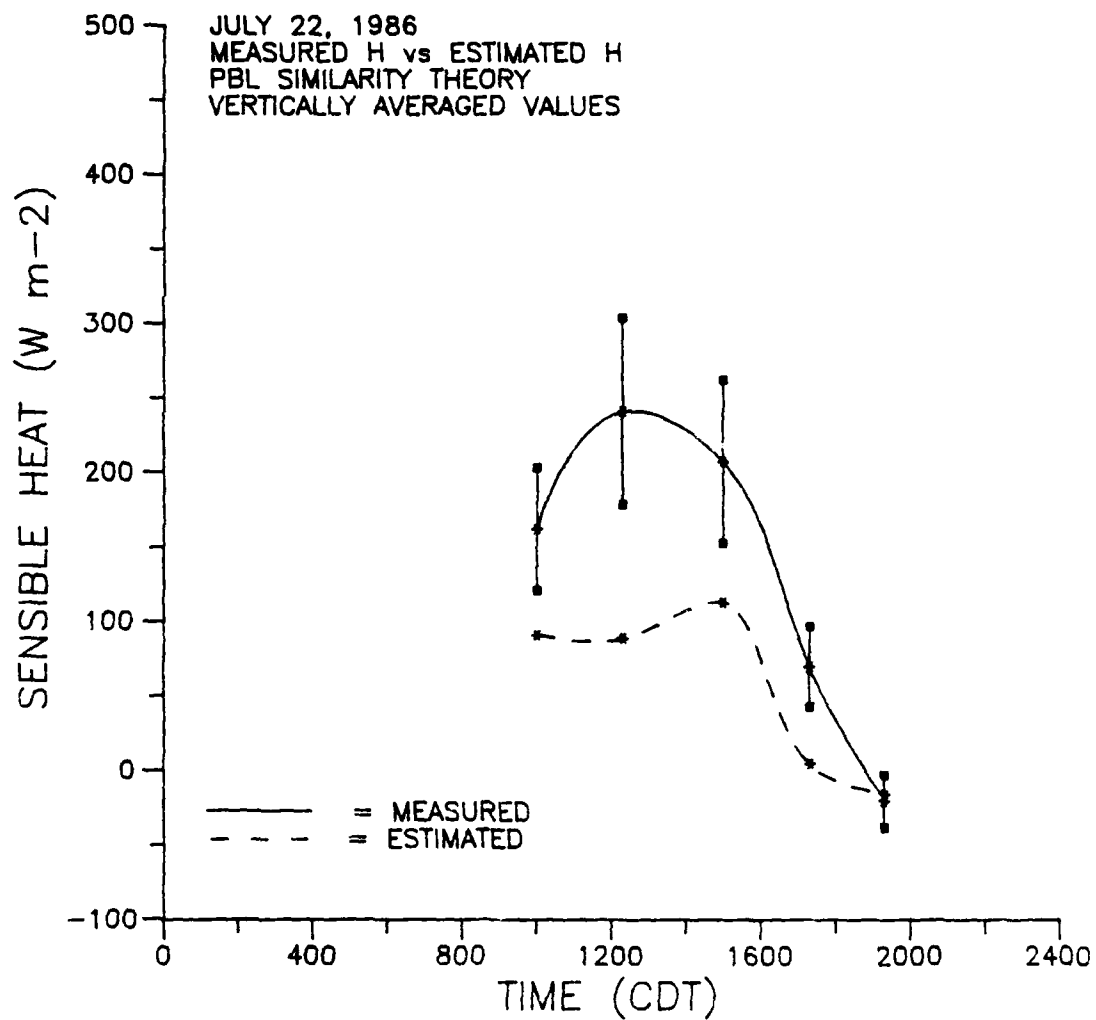


Fig. 17. PBL Similarity Theory estimates of sensible heat (H) using vertically averaged values versus measured values for July 22, 1986. Vertical lines are one standard deviation from the measured values.

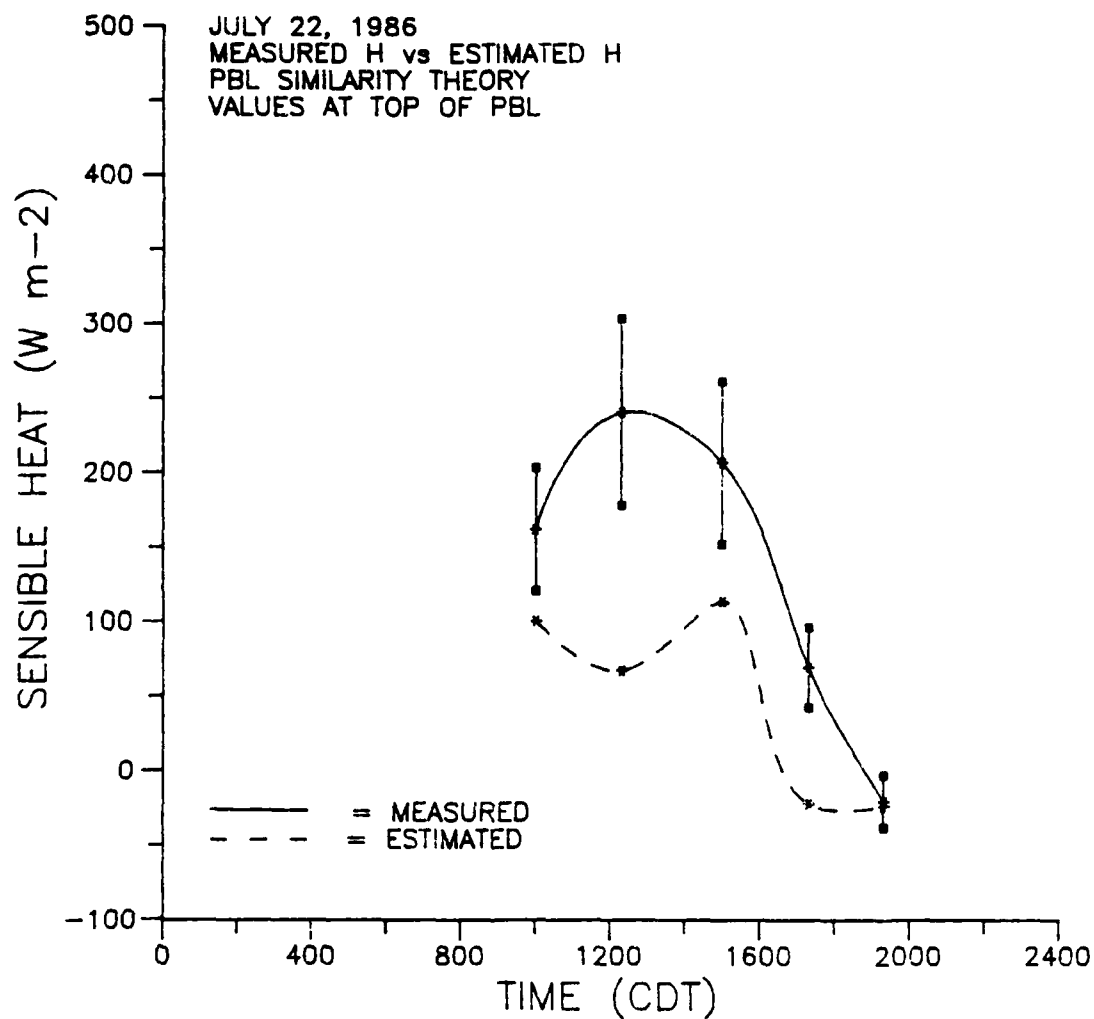


Fig. 18. PBL Similarity Theory estimates of sensible heat ( $H$ ) using values at the top of the PBL versus measured values for July 22, 1986. Vertical lines are one standard deviation from the measured values.

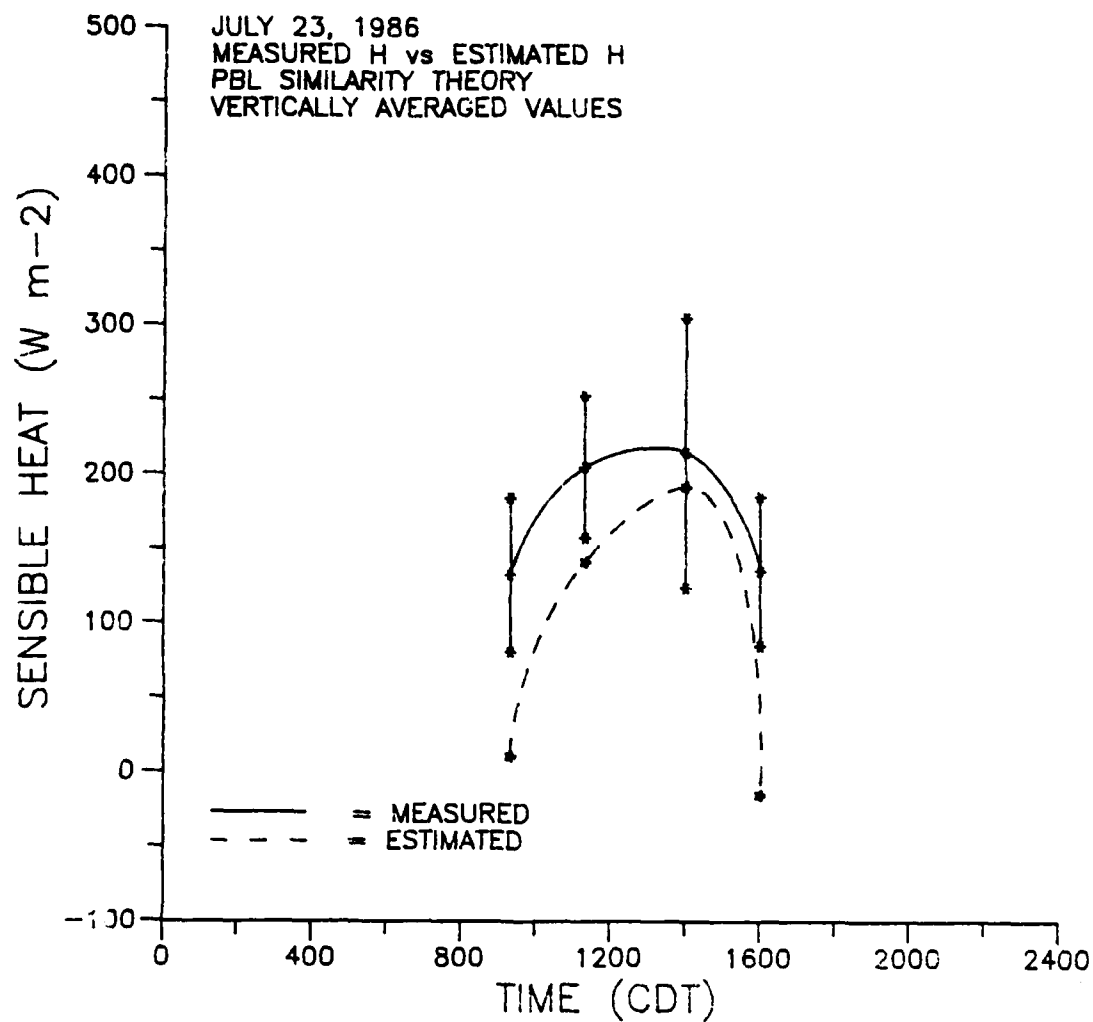


Fig. 19. PBL Similarity Theory estimates of sensible heat ( $H$ ) using vertically averaged values versus measured values for July 23, 1986. Vertical lines are one standard deviation from the measured values.

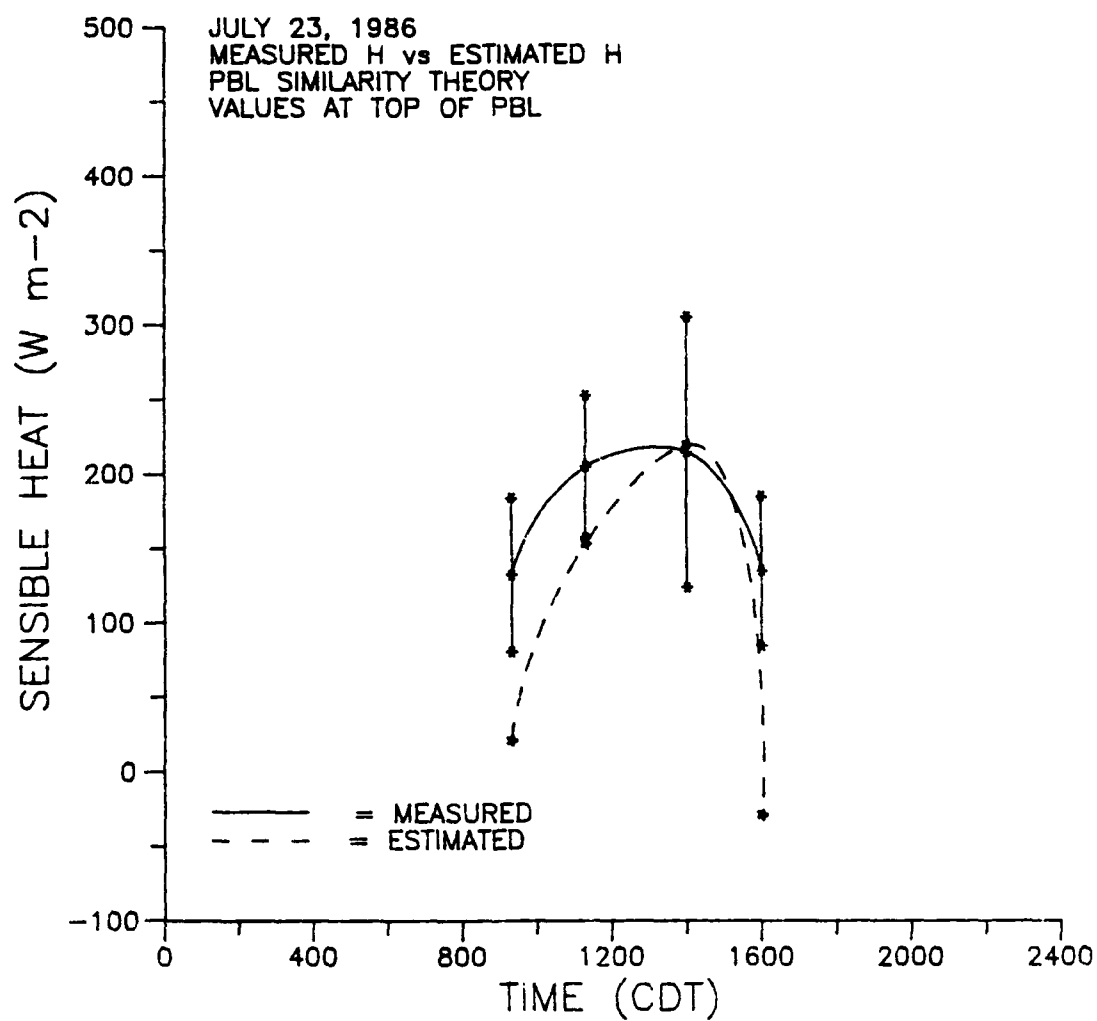


Fig. 20. PBL Similarity theory estimates of sensible heat ( $H$ ) using values at the top of the PBL versus measured values for July 23, 1986. Vertical lines are one standard deviation from the measured values.

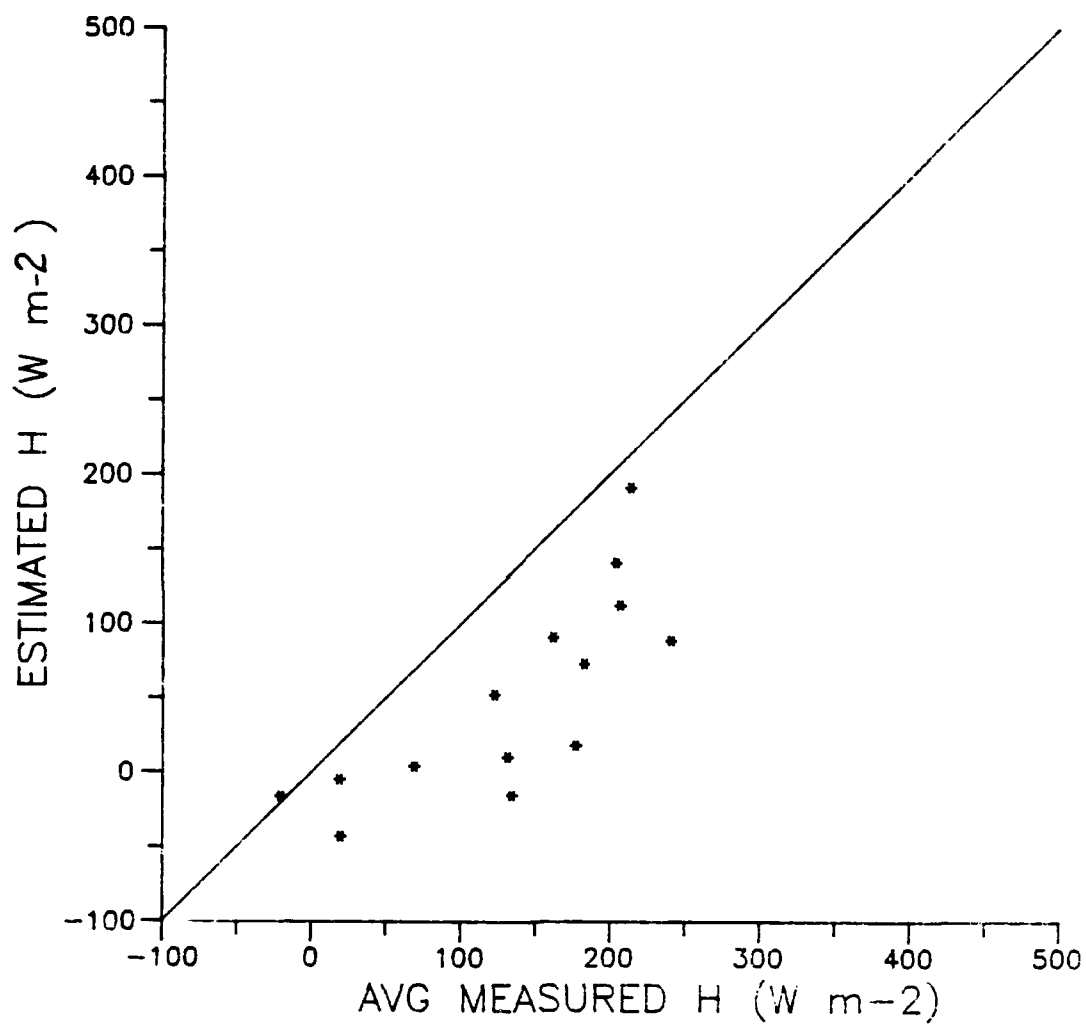


Fig. 21. PBL Similarity Theory estimates of sensible heat ( $H$ ) using vertically averaged values versus measured values for all soundings.

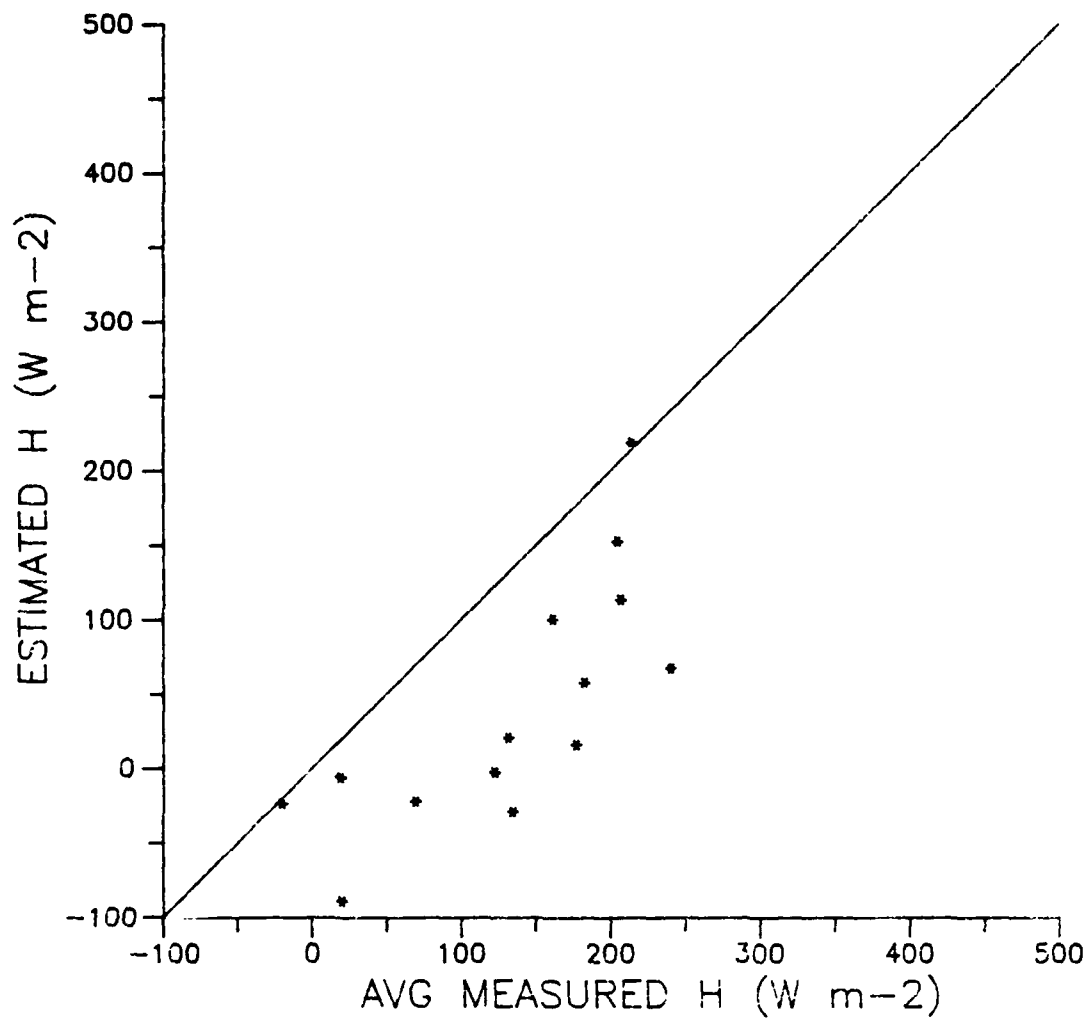


Fig. 22. PBL Similarity Theory estimates of sensible heat (H) using values at the top of the PBL versus measured values for all soundings.

Average deviations for sensible heat flux were  $96.4 \text{ W m}^{-2}$  using values at the top of the PBL and  $107.8 \text{ W m}^{-2}$  using the vertically averaged values, where W is watts and m meters.

#### Latent Heat Flux

Latent heat flux estimates agreed better with surface values for the PBL Similarity Theory using values at the top of the PBL. This method yielded values within one standard deviation for six of the fourteen soundings versus five using vertically averaged values. This may not seem like a significant difference. However, a more instructive way to examine the results is to report the difference between each estimate and the average surface measurements in terms of number of standard deviations. Table 4 presents the estimates by number of standard deviations from the measured values. It is clear from the results that using values at the top of the PBL yields better results for July 21 and 22, but vertically averaged values produced better agreement for July 23 and 24..

Figs. 23-26 illustrate the daily comparisons of the calculated estimates of latent heat flux versus the ground truth data. These figures also show the much better estimates from values at the top of the PBL for July 22, but a slightly reversed situation for July 23.

The average deviations were  $106 \text{ W m}^{-2}$  for values at the top of the PBL and  $124.1 \text{ W m}^{-2}$  using vertically averaged values. Figures 27 and 28 present estimates of latent heat flux for each sounding versus the ground truth data on a 1:1 plot. These plots show that except for two outlying points, both methods provide reasonable estimates.

Table 4. Difference between ground truth data and calculations of sensible and latent heat flux for each approach expressed in number of standard deviations

| DATE/<br>TIME | CONSERVATION<br>EQUATION<br>APPROACH |       | PBL SIMILARITY<br>THEORY AVERAGED<br>VALUES |       | PBL SIMILARITY<br>THEORY VALUES<br>AT TOP OF PBL |       |
|---------------|--------------------------------------|-------|---|-------|--|-------|
|               | H                                    | LE    | H   | LE    | H  | LE    |
| JULY 21       |                                      |       |   |       |  |       |
| 1530          | -4.07                                | -2.40 | -4.74                                       | -2.03 | -4.80  | -1.48 |
| 1900          | .57                                  | -.64  | -.98  | -2.02 | -1.04  | -1.83 |
| JULY 22       |                                      |       |   |       |  |       |
| 1000          | -1.02                                | -.74  | -1.71                                       | -2.03 | -1.48  | -1.06 |
| 1230          | .28                                  | -.20  | -2.42                                       | -1.78 | -2.76  | -.77  |
| 1500          | .66                                  | -.04  | -1.72                                       | -1.15 | -1.70  | -.03  |
| 1730          | -2.55                                | .41   | -2.40                                       | -1.98 | -3.37  | .42   |
| 1930          | 1.11                                 | 3.26  | .24   | -.68  | -.42   | .30   |
| JULY 23       |                                      |       |   |       |  |       |
| 0930          | 1.85                                 | 2.30  | -2.34                                       | 3.93  | -2.14  | 5.26  |
| 1130          | 4.19                                 | 4.37  | -1.33                                       | .95   | -1.08  | 1.14  |
| 1400          | -1.32                                | -1.12 | -.25  | .85   | .05  | 1.71  |
| 1600          | -.44                                 | .24   | -2.97                                       | -.71  | -3.25  | .51   |
| JULY 24       |                                      |       |   |       |  |       |
| 0830          | 7.78                                 | -2.79 | -3.01                                       | .21   | -5.21  | 3.85  |
| 1030          | 1.56                                 | -.80  | -1.37                                       | 1.68  | -2.44  | 3.12  |
| 1200          | .72                                  | -2.00 | -1.46                                       | -1.53 | -1.66  | -.20  |



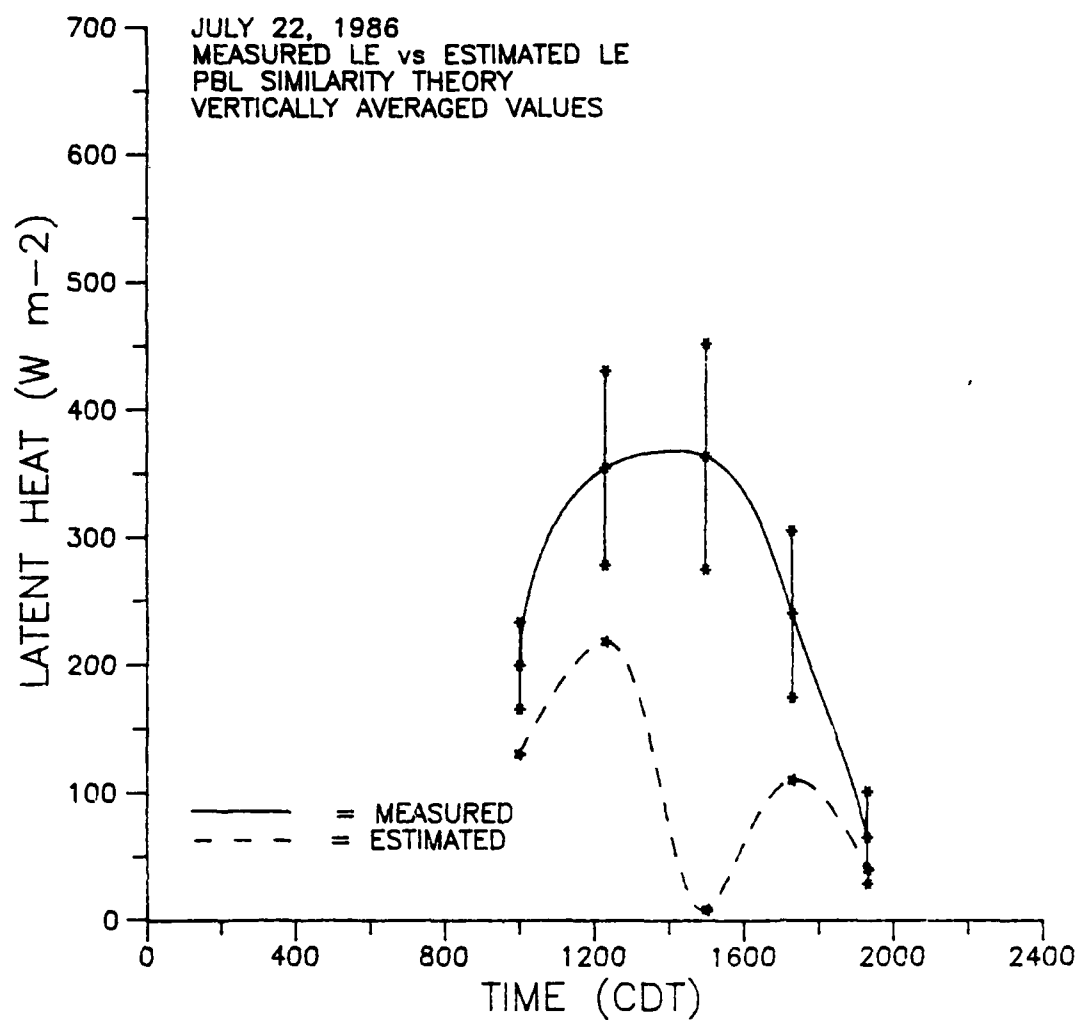


Fig. 23. PBL Similarity Theory estimates of latent heat (LE) using vertically averaged values versus measured values for July 22, 1986. Vertical lines are one standard deviation from the measured values.

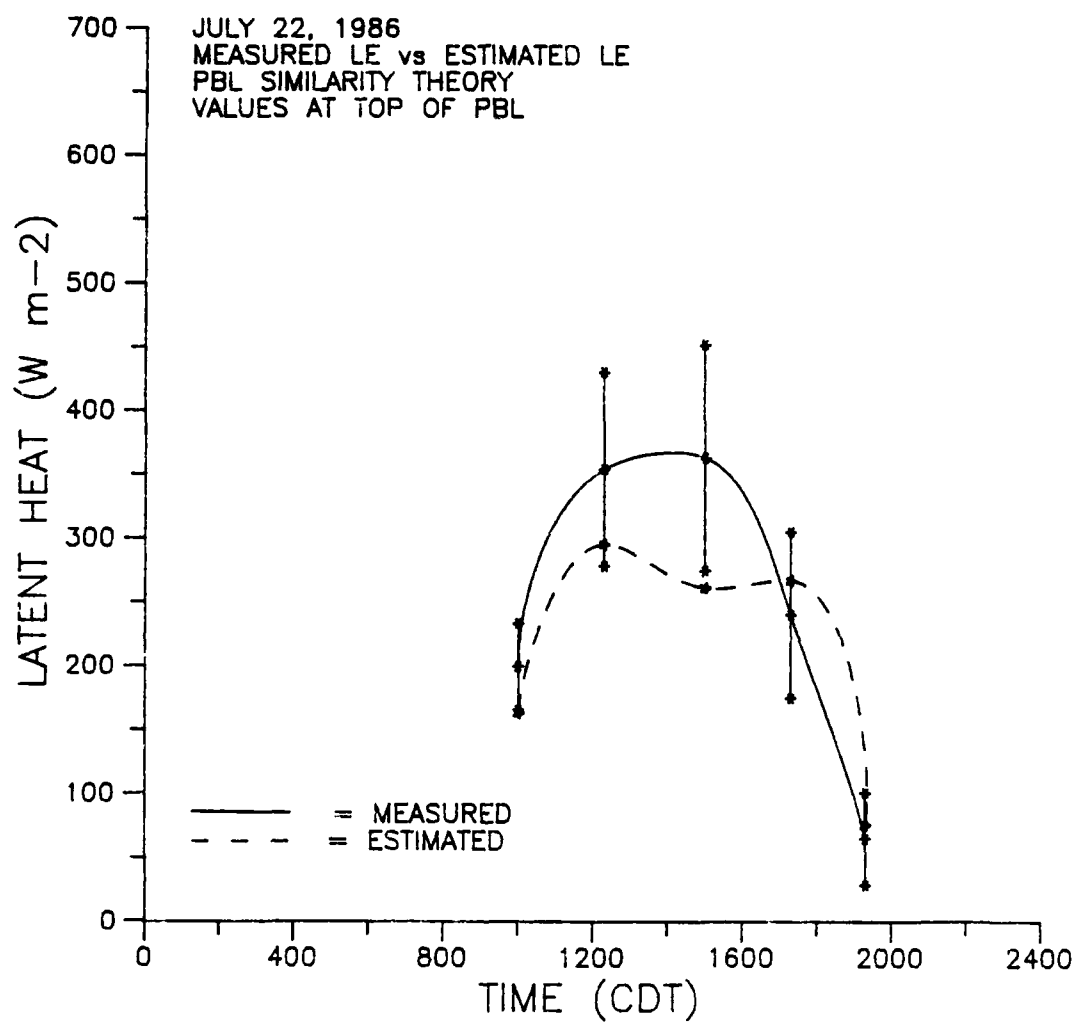


Fig. 24. PBL Similarity Theory estimates of latent heat (LE) using values at the top of the PBL versus measured values for July 22, 1986. Vertical lines are one standard deviation from the measured values.

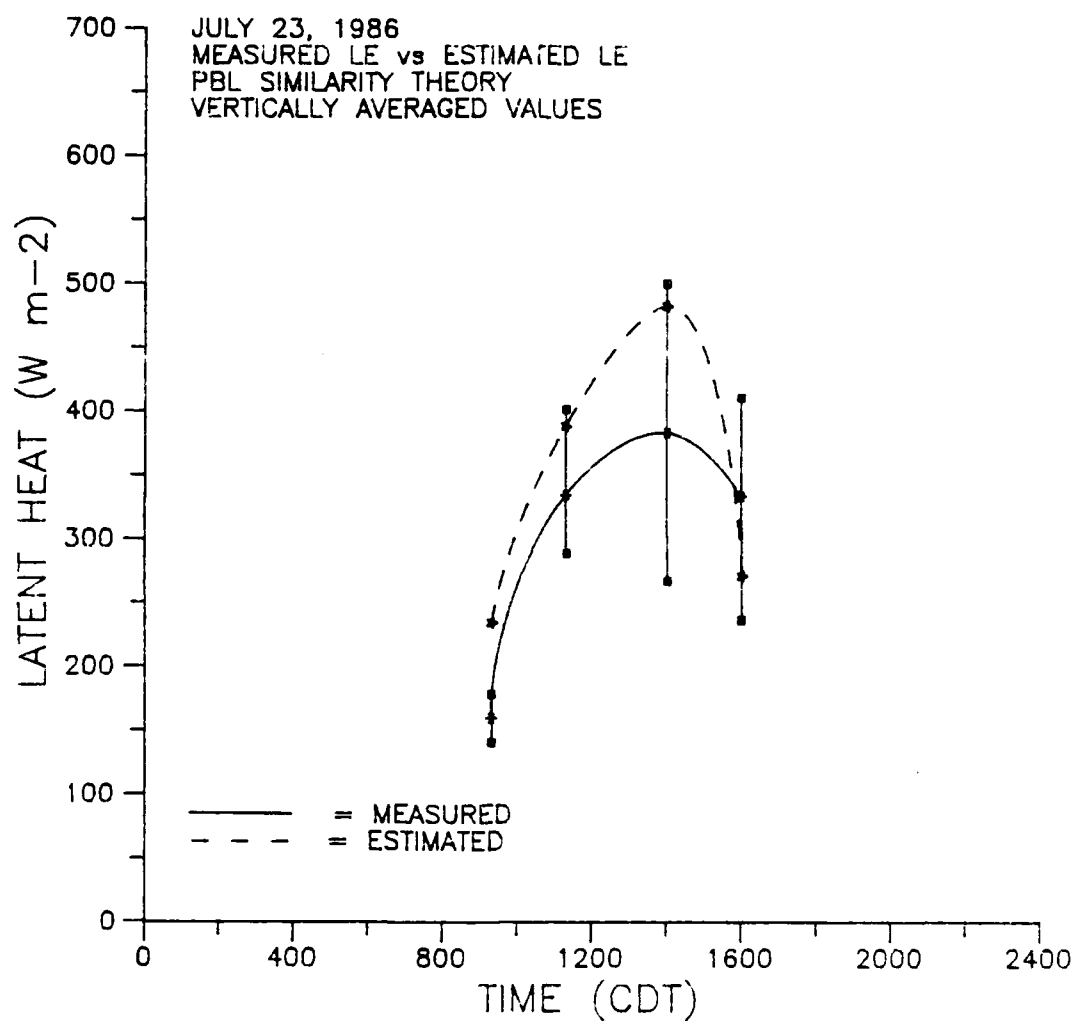


Fig. 25. PBL Similarity Theory estimates of latent heat (LE) using vertically averaged values versus measured values for July 23, 1986. Vertical lines are one standard deviation from the measured values.

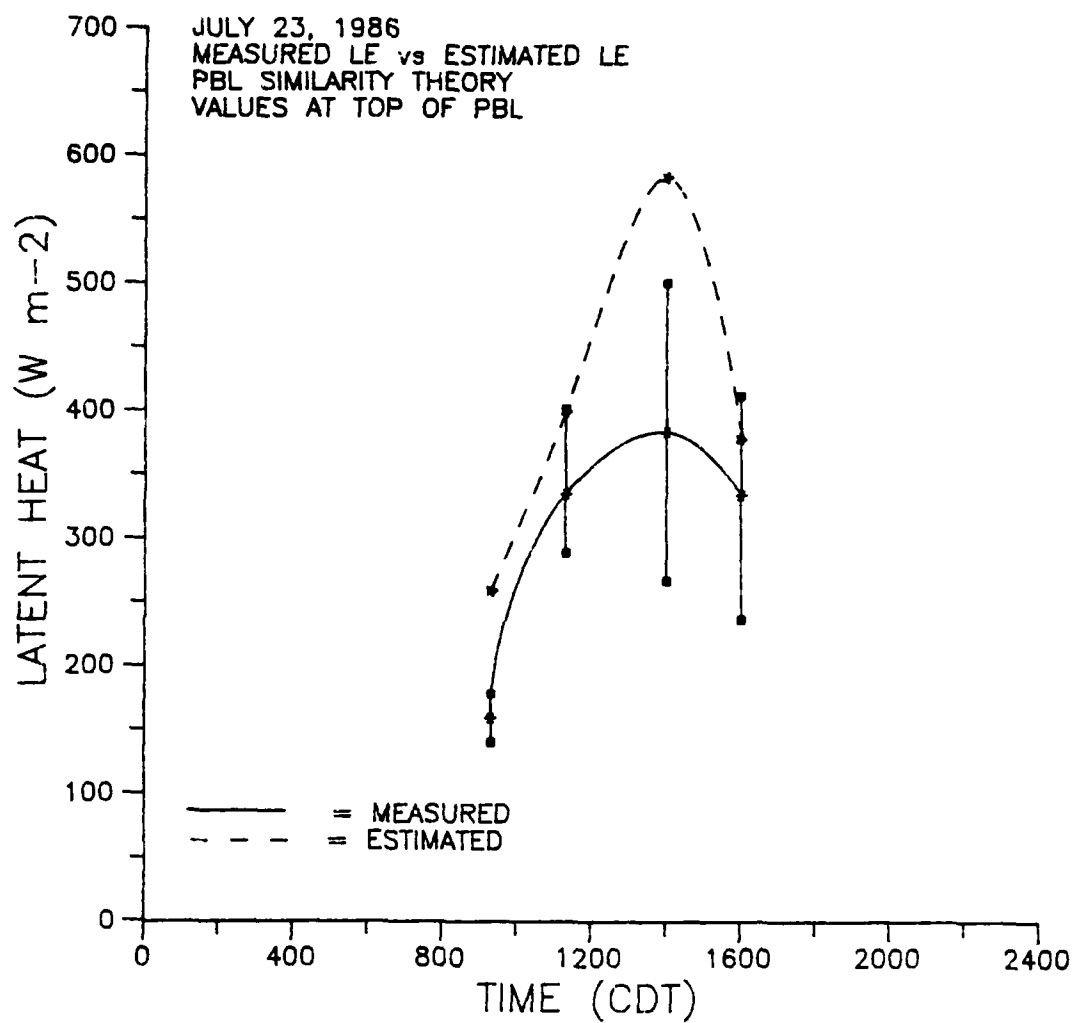


Fig. 26. PBL Similarity Theory estimates of latent heat (LE) using values at the top of the PBL versus measured values for July 23, 1986. Vertical lines are one standard deviation from the measured values.

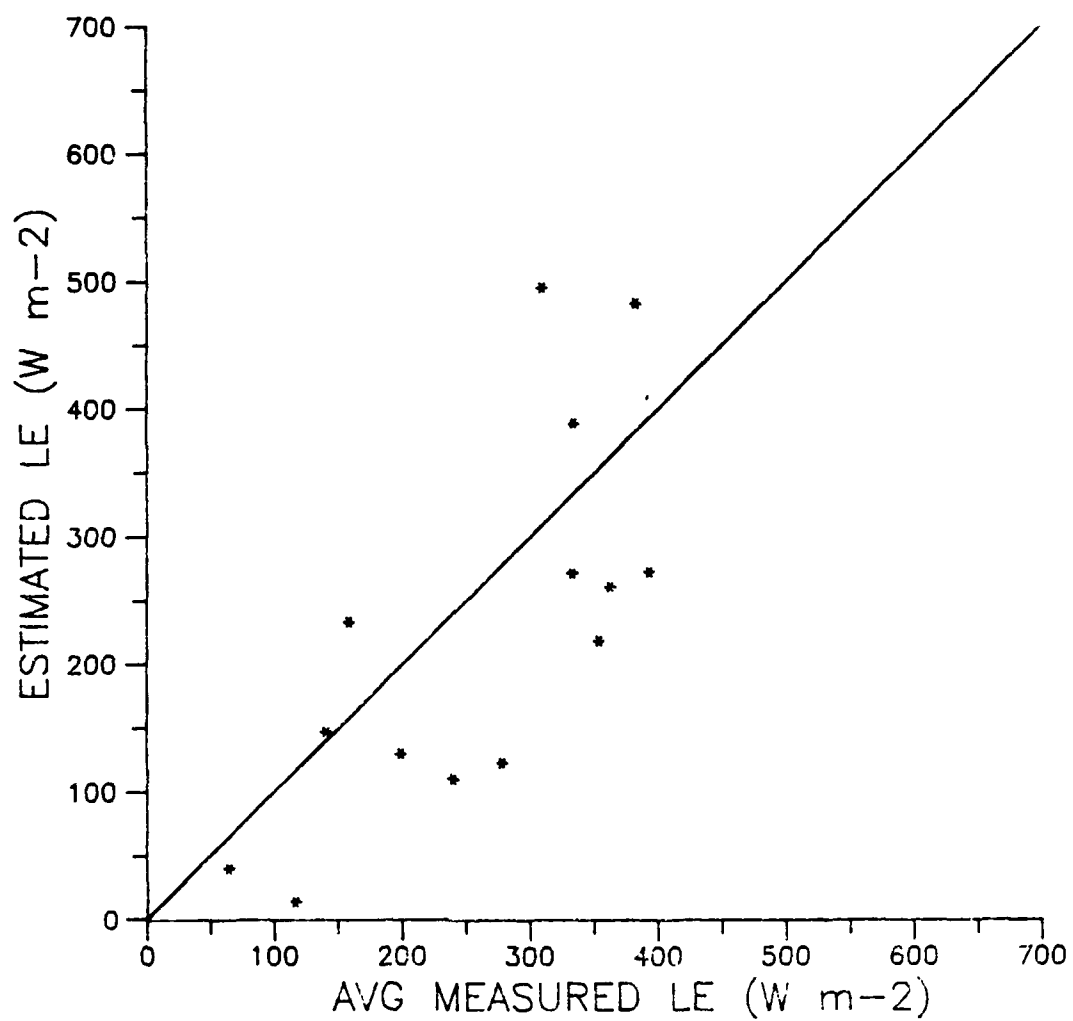


Fig. 27. PBL Similarity Theory estimates of latent heat (LE) using vertically averaged values versus measured values for all soundings.

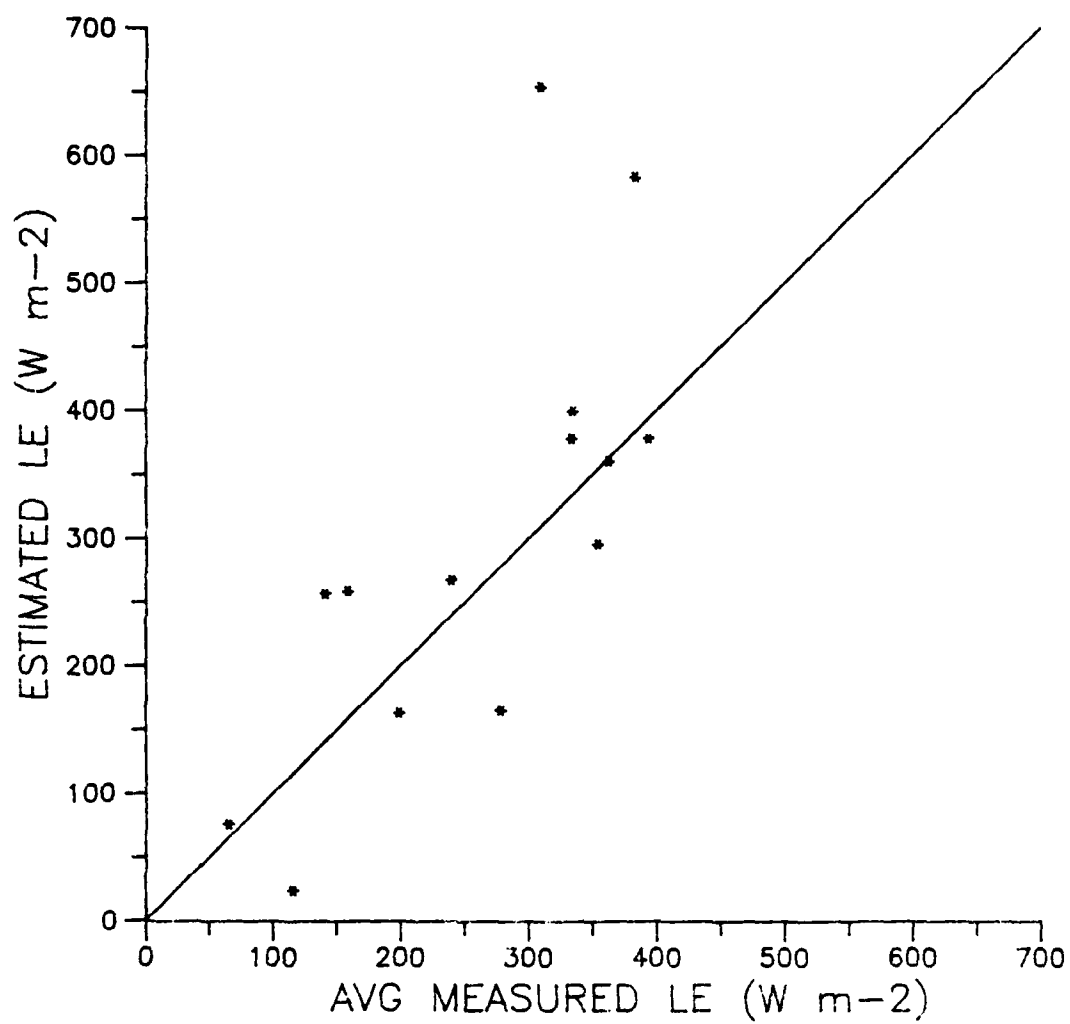


Fig. 28. PBL Similarity Theory estimates of latent heat (LE) using values at the top of the PBL versus measured values for all soundings.

It has been noted that the manually recorded soundings did not yield the same quality estimates as the automatically recorded soundings. A close analysis of Table 4 shows that the agreement for July 22 is really very good. This was the only day that a complete set of soundings using the automatically recorded data was available.

#### CONSERVATION EQUATION APPROACH

The conservation equations of the Mixed Layer PBL model proposed by McNaughton and Spriggs (1986) produced estimates for latent heat flux close to the ground truth measurements for all soundings except the final sounding for July 22. Agreement for sensible heat flux was not as good.

As described earlier, sensible and latent heat fluxes were estimated using equations (42-43). Figs. 29-32 show daily comparisons of the calculated estimates versus the averages from the nine Bowen Ratio systems for 22 and 23 July. Figs. 29 and 30 for July 22 show the best agreement using this approach. Estimates fall within one standard deviation of the measurement values for four of the five soundings for both sensible and latent heat. The other sounding is within two standard deviations. The soundings for 23 July seen in Figs. 31-32 do not exhibit as close an agreement. Even though most of the points do not fall within one standard deviation, most of them do fall in the two standard deviation range.

The good correlation for July 22 is likely due to the fact that those soundings included all measured data points. This being the case, the soundings were much smoother, selection of the inversion height and the base of the mixed layer were much more accurate, and a

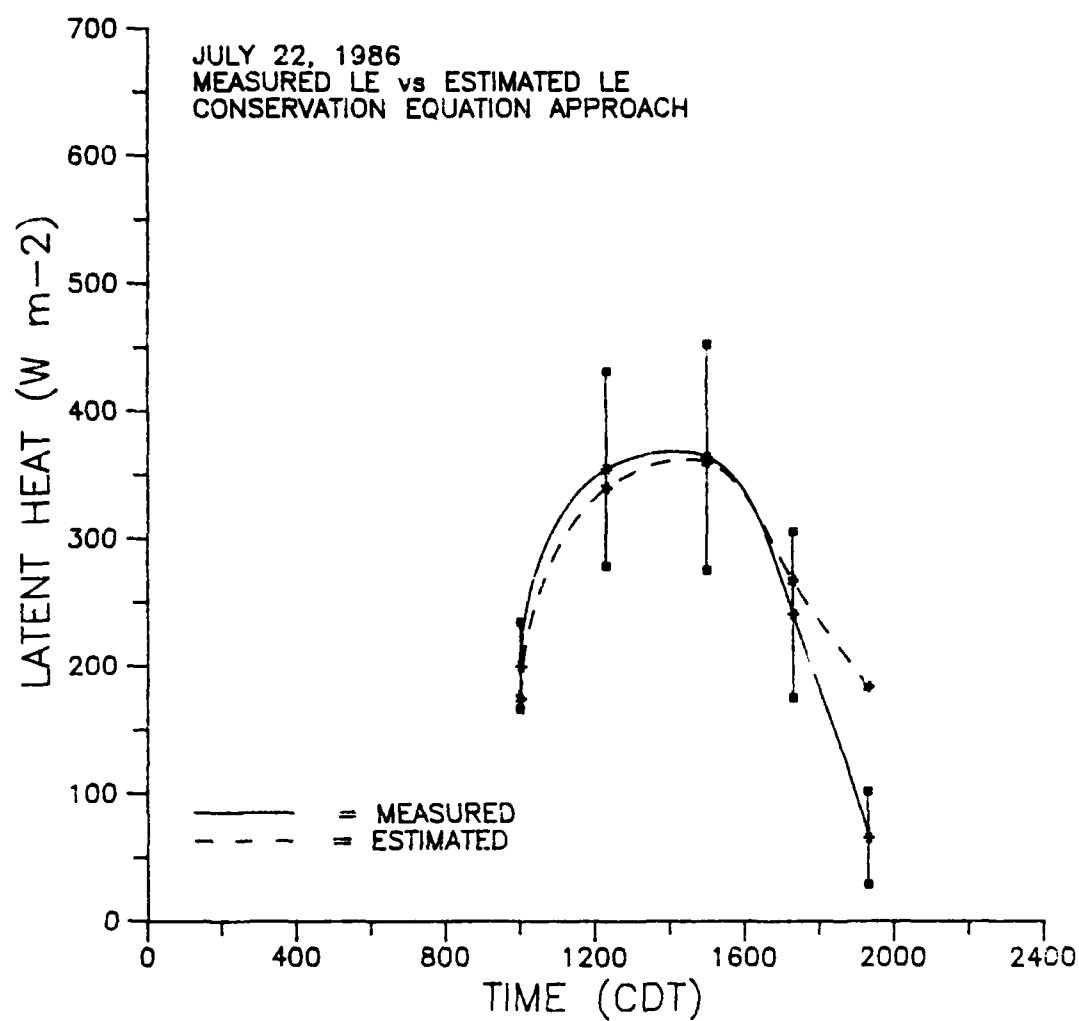


Fig. 29. Conservation Equation Approach estimates of latent heat (LE) versus measured values for July 22, 1986. Vertical lines are one standard deviation from the measured values.



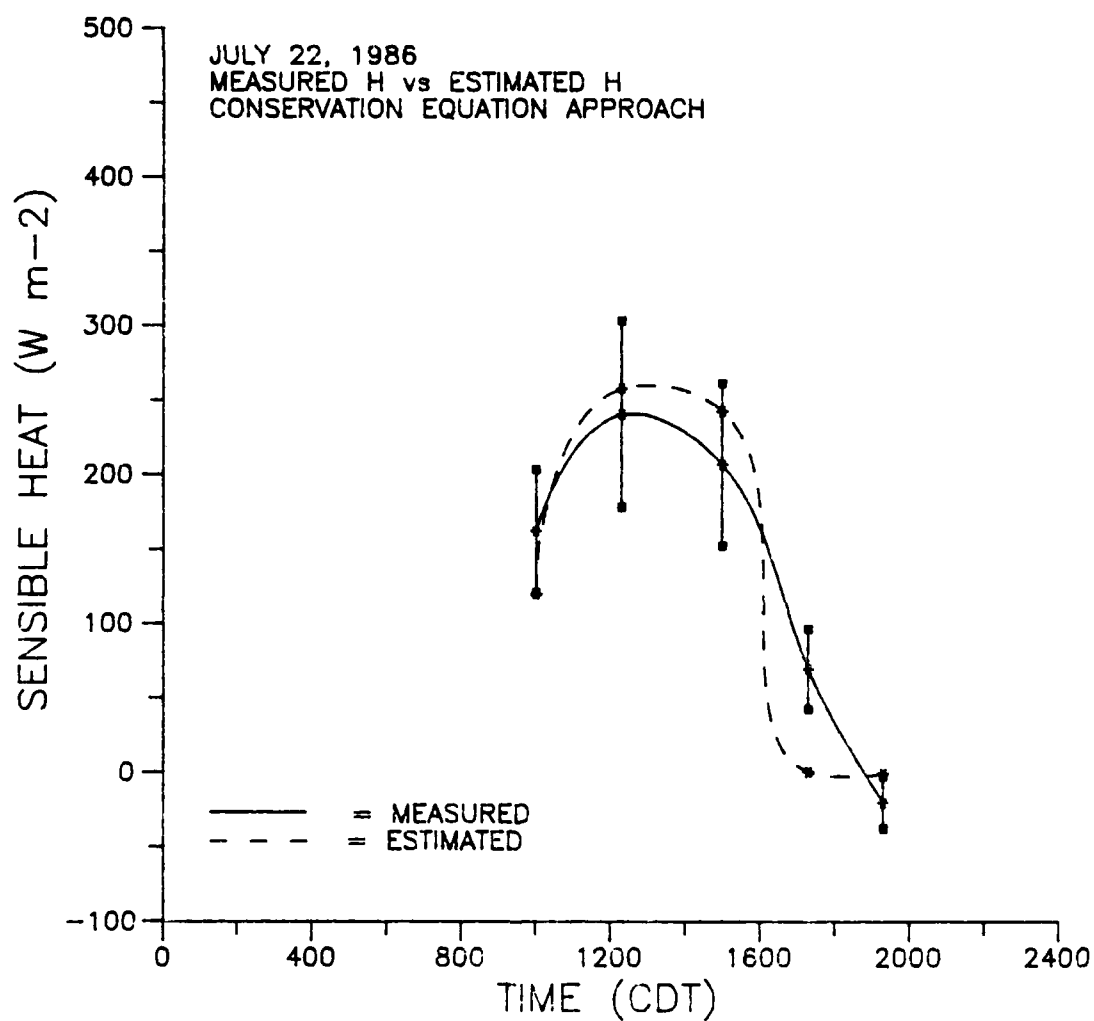


Fig. 30. Conservation Equation Approach estimates of sensible heat (H) versus measured values for July 22, 1986. Vertical lines are one standard deviation from the measured values.

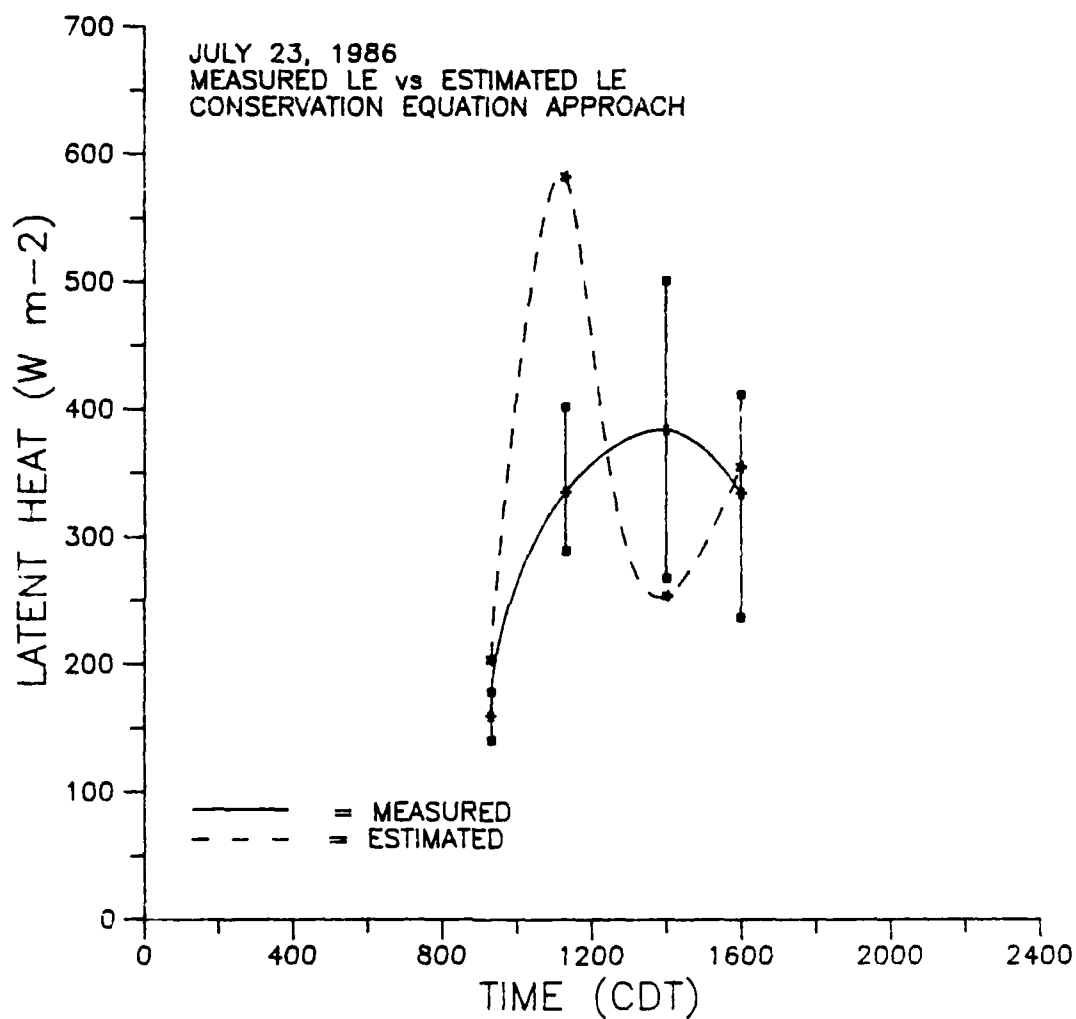


Fig. 31. Conservation Equation Approach estimates of latent heat (LE) versus measured values for July 23, 1986. Vertical lines are one standard deviation from the measured values.

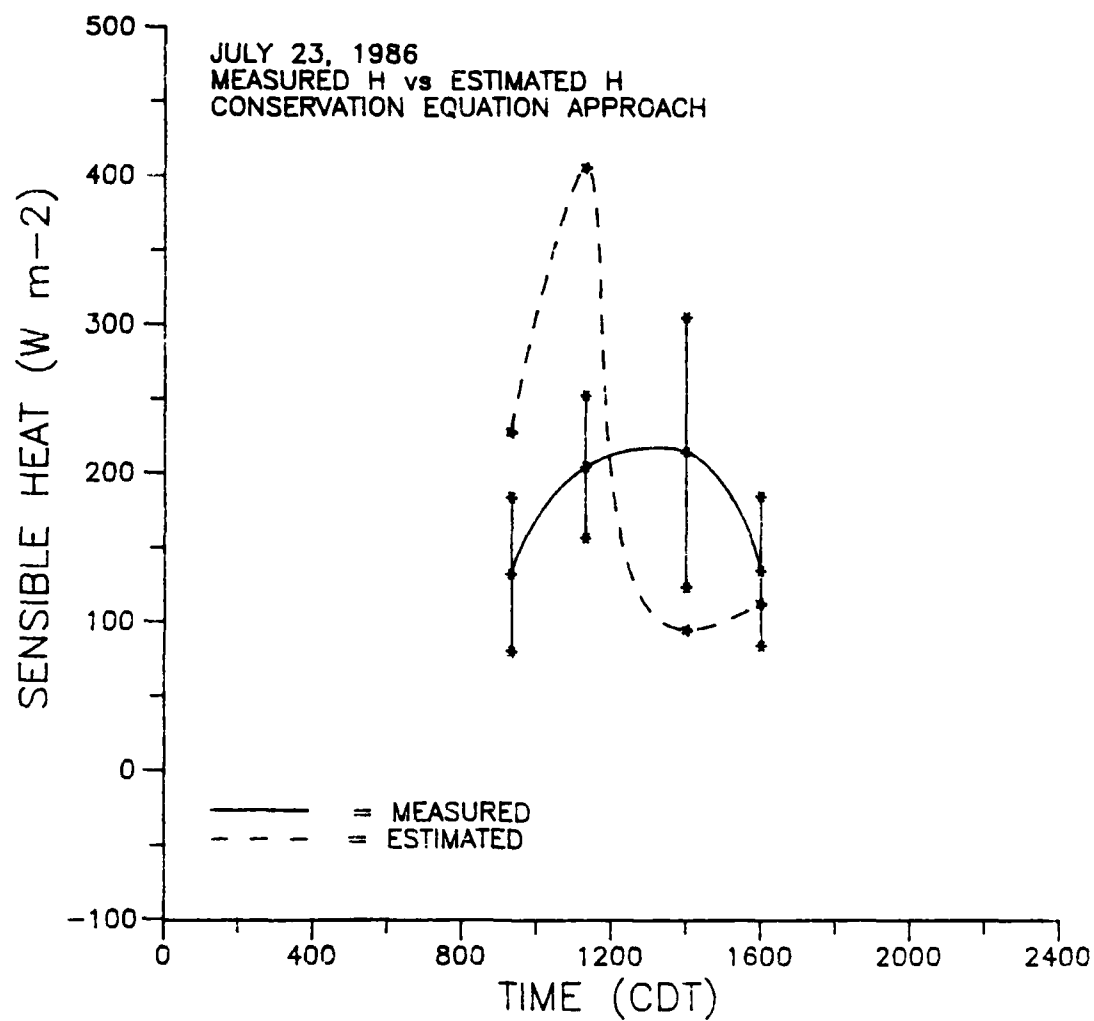


Fig. 32. Conservation Equation Approach estimates of sensible heat (H) versus measured values for July 23, 1986. Vertical lines are one standard deviation from the measured values.

better average over the mixed layer was obtained. However, the soundings for July 23 used only the 30-second data, causing the determination of the inversion height to be much more difficult and less accurate. This lead to the less reliable estimates of both sensible and latent heat flux.

Figs. 33-34 present the results of this approach against a 1:1 line for both sensible and latent heat. Most of the estimated values show a close relationship to the measured values. The average deviation was  $95.5 \text{ W m}^{-2}$  and  $90.6 \text{ W m}^{-2}$  for sensible and latent heat, respectively. Table 4 shows the estimates represented by the number of deviations from the measured values. The standard deviation of the measured data ranges from  $\pm 20$  to  $\pm 117 \text{ W m}^{-2}$ , thus a lot of variation exists in the ground truth estimates.

In general, both the PBL Similarity Theory using values at the top of the PBL and the Conservation Equation Approach yield reasonable estimates of regional sensible and latent heat flux as long as the full soundings are available and an accurate inversion height can be determined. Such is the case for July 22.

It must also be noted that since the Conservation Equation Approach uses the rate of change between two soundings for its calculations, one bad sounding will cause two estimates to be distorted, not just one.

The Conservation Equation Approach might be expected to do better than the PBL Similarity Theory since it simply involves solving the conservation equations using the measured data. The major weakness is that entrainment is neglected. The PBL Similarity Theory, on the other hand, deals with the empirically derived similarity parameters and many assumptions, which makes the theory less reliable under different

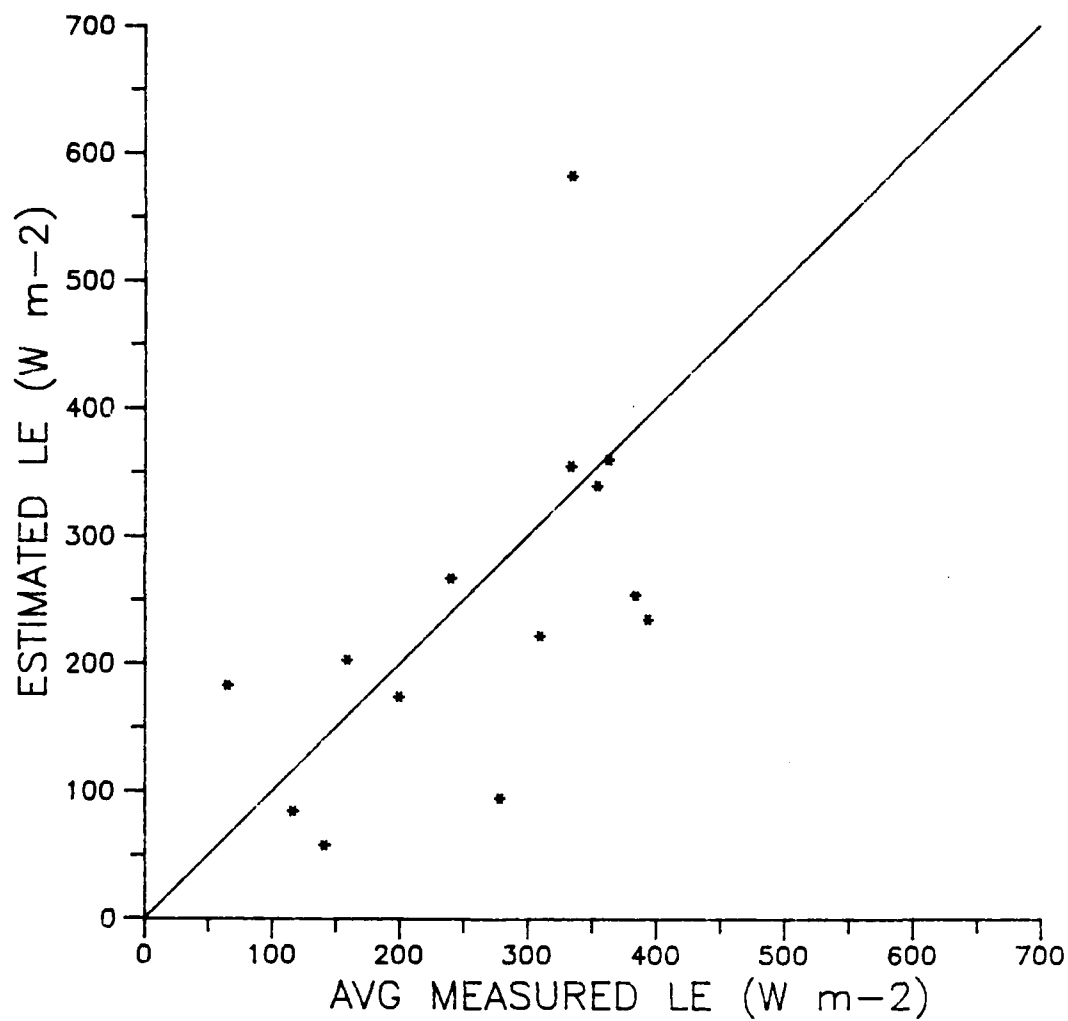


Fig. 33. Conservation Equation Approach estimates of latent heat (LE) versus measured values for all soundings.

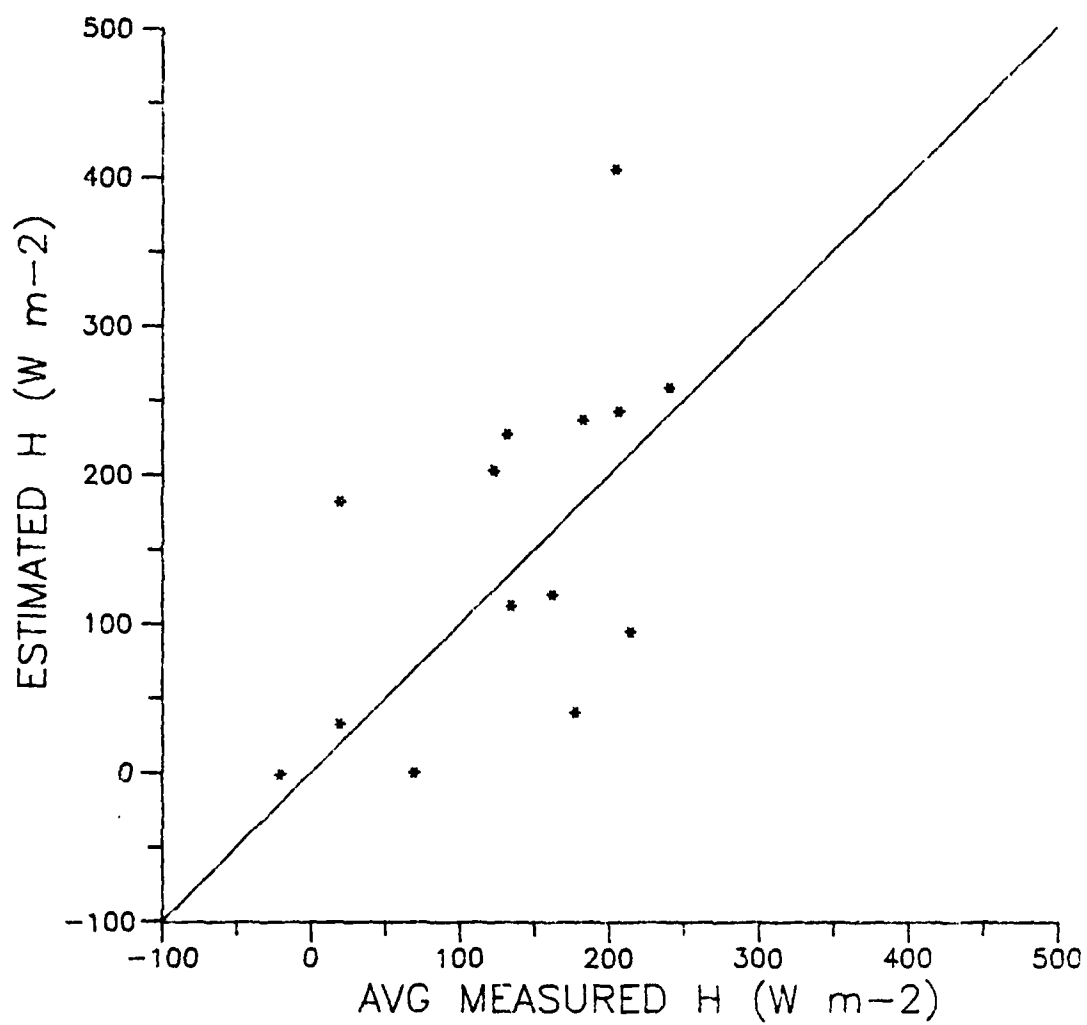


Fig. 34. Conservation Equation Approach estimates of sensible heat (H) versus measured values for all soundings.

stability conditions. The data show that under stable conditons the PBL Similarity Theory yields much lower values. The PBL Similarity Theory also requires that surface values be interpolated from the data. This empirical determination of surface values is another weakness of this theory, as compared to the Conservation Equation Approach.

Overall, the conservation Equation Approach gives the most acceptable estimates of regional evapotranspiration and sensible heat flux. Except for one sounding on July 22 all estimates are within reasonable range of "error" for such an approach, given the large variation in the surface measurements. This is clearly illustrated in Table 4.

## SUMMARY AND CONCLUSIONS

Two theoretical approaches for estimating regional fluxes of sensible and latent heat were introduced and tested. Values from these approaches were compared against ground truth measurements from nine Bowen Ratio systems. This work represents a reasonably rigorous experiment to estimate regional fluxes and compare them to a fairly large array of surface measurements.

Data were collected for a 3 1/2 day period over the Konza Prairie Natural Area, south of Manhattan, Kansas, from 21-24 July 1986. The processed data were applied to Planetary Boundary Layer Similarity Theory in two separate approaches and to the conservation equations for heat and water vapor. Both approaches yielded acceptable estimates for both sensible and latent heat in most cases.

Better results were obtained when the virtual potential temperature soundings were plotted using the automatically recorded data (every five seconds). It is important to note that since determination of the inversion height appears to be so critical to the results, automatically recorded data are preferred. Soundings using 30-second manually recorded data may or may not give a detailed enough sounding to determine the inversion height accurately.

The height of the PBL ranged from 275 meters during the morning and up to 1700 meters when fully developed in the afternoon.

PBL Similarity Theory yielded mixed results. Although using values from the top of the PBL gave a deviation between estimated and measured values for latent heat of  $106 \text{ W m}^{-2}$ , and the deviation using the averaged values was  $124.1 \text{ W m}^{-2}$ , no distinction was made as to



which method was better. This was due to the fact that neither method was exceedingly better than the other for any one day. Similar results were found for sensible heat. Deviations for July 22 (Table 4) show that if a good sounding is available and an inversion height can be determined, good estimates of both sensible and latent heat can be calculated using both theoretical approaches.

The Conservation Equation Approach gave the better estimates. The fact that this approach simply solves the conservation equations for heat and humidity and is not affected by the stability of the atmosphere makes this the favored approach.

Continued work is required using both approaches. As is shown by these preliminary results, a small number of soundings will cause higher than expected average deviations if only one sounding is in error.

A larger-scale experiment over several years is necessary to truly evaluate these approaches and to validate these preliminary results. A large data base would allow obviously erroneous soundings to be discarded without biasing the results. Measurements made through all four seasons would show whether the theories work during all types of weather conditions. The most important part of such a large-scale experiment would be to ensure that a wide array of ground truth measurements for verification of the estimates is available. Without the ground measurements, justification of the theoretical approaches is impossible.

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